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THE PSYCHOPHYSICAL LAW AND FUNCTIONAL MEASUREMENT

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ABSTRACT

Rather than focusing on the psychophysical law—an arbitrary law— the functional measurement approach to psychophysics focuses on three objectives: to measure conscious sensation, to determine the rule of integration of the components of information that produce the conscious sensation, and to measure such components.

In any psychophysical law—for example, in Fechner's law ($\psi = a \log \phi + b$, 1860) or in Plateau's law ($\psi = c\,\phi^d$, 1872)—sensory intensity ψ is related to physical intensity ϕ arbitrarily.

Arbitrary ϕ. Generally, sensations are multidetermined (Marks, 1978). Surface brightness, for example, is determined simultaneously by the luminance of the surface and by that of the background, and heaviness is determined simultaneously by object weight and by object volume. In these cases, the psychophysical law is arbitrary because the choice of ϕ is arbitrary.

Arbitrary definition of ϕ. Different definitions of ϕ may be used equivalently for the psychophysical law (Myers, 1982). For example, odorant concentration varies with the number x of molecules of odorant and with the number y of molecules of air in the mixture. In different studies of olfactory intensity, ϕ has been defined either as $\phi' = x / y$ or as $\phi'' = x / (x + y)$. Physically, these definitions are equivalent because they are equally informative. However, they influence the psychophysical law differently. For example, consider Plateau's law with ϕ' or ϕ'', that is, $\psi = e\,(\phi')^f$ or $\psi = g\,(\phi'')^h$. Plateau's law is a power function for both ϕ' and ϕ'' only if $\phi'' = m\,(\phi')^n$ (Myers, 1982). It follows that, if Plateau's law is a power function for ϕ', this law is not a power function for ϕ'' because $\phi'' = \phi' / (1 + \phi') \neq m\,(\phi')^n$. When there are different equivalent definitions of ϕ, the psychophysical law is arbitrary because the choice of one of these definitions is arbitrary.

Arbitrary scale of ϕ. For the same physical variable, different empirical operations allow to construct different physical scales each satisfying all of the axioms required for scale construction (Ellis, 1966; Falmagne, 1985). Consequently, physical scales are arbitrary. It follows that any psychophysical law is arbitrary because the choice of the physical scale for ϕ is arbitrary.

Even if they are arbitrary, psychophysical laws are important in practice. They serve to summarize sensory data. For example, astronomers define apparent stellar magnitude using Fechner's law and colorists define the value of Munsell gray papers using Plateau's law.

		Factor ϕ_w					
		ϕ_{w_1}	ϕ_{w_2}	…	ϕ_{w_j}	…	ϕ_{w_J}
	ϕ_{v_1}	R_{11}	R_{12}	…	R_{1j}	…	R_{1J}
	ϕ_{v_2}	R_{21}	R_{22}	…	R_{2j}	…	R_{2J}
Factor ϕ_v	…	…	…	…	…	…	…
	ϕ_{v_i}	R_{i1}	R_{i2}	…	R_{ij}	…	R_{iJ}
	…	…	…	…	…	…	…
	ϕ_{v_I}	R_{I1}	R_{I2}	…	R_{Ij}	…	R_{IJ}

Table 1. Real number R_{ij} into which the self-evaluation of heaviness of an object with one of I fixed volumes ϕ_{v_i} and one of J fixed weights ϕ_{w_j} is transduced.

The above observation that sensations are multidetermined indicates that the objective of psychophysics is vaster than the objective of establishing relations between sensory and physical intensities. Usually, multidetermined sensations are studied in factorial experiments using some method of self-evaluation. In a factorial experiment on heaviness, for example, subjects lift and simultaneously self-evaluate the heaviness ρ_{ij} of single objects with volume ϕ_{v_i}, $i = 1, 2, ..., I$, and with weight ϕ_{w_j}, $j = 1, 2, ..., J$. Each self-evaluation is transduced into a real number R_{ij}. Table 1 represents R_{ij} for each pair of ϕ_{v_i} and ϕ_{w_j}. The general finding that R_{ij} varies with both ϕ_{v_i} and ϕ_{w_j} (Anderson, 1970; Stevens & Rubin, 1970) shows that ρ_{ij} results from the integration of some component s_{v_i} of visual information about object volume and of some component s_{w_j} of muscular information about object weight.

This analysis shows three objectives for psychophysics: to measure conscious sensation (ρ_{ij}), to determine the rule of integration of the components of information (s_{v_i} and s_{w_j}) that produce the conscious sensation, and to measure such components. How can these objectives be achieved?

The functional measurement approach to psychophysics answers this question as follows (Anderson, 1981, 1996): assume that the rating method provides measures of sensation on an interval scale and use this method in a factorial experiment knowing the principle that the response function and the integration rule jointly determine the pattern of factorial graphs.

That the rating method provides measures of sensation on a interval scale means that the response function relating R_{ij} and ρ_{ij} is linear. Since context may make this function nonlinear (Parducci, 1982) care must be taken that context effects are minimal. For example, end anchors should be used to reduce floor and ceiling effects, and the response range should be wide.

To illustrate the above principle that the response function and the integration rule jointly determine the pattern of factorial graphs, suppose that the response function is

$$R_{ij} = c_0 + c_1 \rho_{ij}, \quad [1]$$

with c_0 and c_1 constants, and that the integration rule is

$$\rho_{ij} = s_{v_i} + s_{w_j}. \quad [2]$$

Together, Equations 1 and 2 imply that factorial graphs are parallel when R_{ij} is plotted, for example, as a function of ϕ_{w_j}, with ϕ_{v_i} the parameter. In fact, consider R_{ij} in any two rows of Table 1, for example, Rows 1 and 2. For Column *j*, Equation 1 implies that

$$R_{1j} = c_0 + c_1\, \rho_{1j}$$

and

$$R_{2j} = c_0 + c_1\, \rho_{2j}$$

and Equation 2 implies that

$$R_{1j} = c_0 + c_1\, (s_{v_1} + s_{w_j})$$

and

$$R_{2j} = c_0 + c_1\, (s_{v_2} + s_{w_j}).$$

The difference

$$R_{1j} - R_{2j} = c_1\, (s_{v_1} - s_{v_2})$$

is constant for each ϕ_{w_j}, which shows that factorial graphs must be parallel.

The assumption of linearity of the response function has two implications which, if true, are of fundamental importance for psychophysics: (i) the response function measures conscious sensation on an interval scale and, as we have just seen, (ii) observation of the pattern of factorial graphs suffices to determine the integration rule. To make sure that these implications are true, validational tests are required to make sure that the assumption that the rating method implies a linear response function is true. Positive results of these tests constitute converging evidence that ratings measure conscious sensation and that factorial graphs disclose the integration rule. I give two examples of such tests taken from a list of nine (Anderson, 1996, pp. 94-96).

Weiss (1972) used the rating method assuming a linear response function. He prescribed the rule of integration by asking subjects to rate the average grayness of pairs of Munsell chips. Together, response function linearity and this rule imply parallel factorial graphs (Anderson, 1981). The obtained factorial graphs were parallel indicating that the response function was linear.

Weiss counterchecked his results as follows. Curtis, Attneave, & Harrington (1968) demonstrated that magnitude estimation implies a nonlinear response function. If the factorial graphs are parallel because the rating response function is linear, then factorial graphs must be nonparallel when subjects estimate the magnitude of the average grayness of pairs of Munsell chips. Weiss (1972) had subjects produce such magnitude estimations. With such estimations, factorial graphs were nonparallel confirming the linearity of the rating response function.

Before we see the second example we need to see how components of sensory information can be measured.

For each Column *j* of Table 1 consider the mean

$$\overline{R}_{.j} = \Sigma\, R_{ij} \,/\, I.$$

Equation 1 implies that

$$\overline{R}_{.j} = \Sigma\, (c_0 + c_1\, \rho_{ij}) \,/\, I,$$

that is,

$$\overline{R}_{.j} = c_0 + c_1 \Sigma \rho_{ij} / I.$$

Equation 2 implies that

$$\overline{R}_{.j} = c_0 + c_1 (\Sigma s_{v_i} + \Sigma s_{w_j}) / I.$$

Since $c_1 \Sigma s_{v_i} / I$ is a constant and s_{w_j} is the same for each row,

$$\overline{R}_{.j} = c_0' + c_1 s_{w_j}$$

with c_0' a constant equal to $c_0 + c_1 \Sigma s_{v_i} / I$.

That is, the mean $\Sigma R_{ij} / I$ for each Column j is a measure of the component s_{w_j} of muscular information about object weight. Similarly, for each Row i of Table 1, the mean $\Sigma R_{ij} / J$ is a measure of the component s_{v_i} of visual information about object volume (Anderson, 1981). Both measures are on an interval scale.

Thus, in addition to providing validational procedures indicating that R_{ij} measures conscious sensation on an interval scale and that the integration rule may be determined from observation of the pattern of factorial graphs, the functional measurement approach also provides measures of the components of sensory information that determine the conscious sensation.

The second example of a validational test is about whether different tasks produce the same measures of components of sensory information. In the experiment on heaviness mentioned above, where subjects rated heaviness of single lifted objects while they saw the objects, empirical factorial graphs were parallel indicating that the integration rule was additive. When subjects rate average heaviness of two unseen successively lifted objects, empirical factorial graphs are also parallel in agreement with an additive integration rule. In these two tasks, the component s_{w_j} of muscular information about object weight must be the same for each lifted object. That is, measures of s_{w_j} from the two tasks must be related linearly. This linear relation has been confirmed empirically (Anderson, 1972, 1974).

The functional measurement approach has made three fundamental contributions to psychophysics. The first is the demonstration that the rating method provides measures of conscious sensation (for example of ρ_{ij}) on an interval scale. The psychophysical law relates these measures to some physical intensity. The second contribution is the demonstration that the pattern of factorial graphs reveals the nature of the integration rule. Thus, functional measurement extends the original objective of psychophysics, which was that of establishing relations between sensory and physical intensities, to that of constructing theories of sensory processes. The third contribution is the demonstration that the use of the rating method in factorial experiments provides measures on an interval scale of components of information (for example of s_{v_i} and s_{w_j}) that determine the conscious sensation.

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