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TEST OF LINEARITY OF THE RESPONSE FUNCTION FOR RATINGS OF PERCEIVED AREA

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Abstract

The linearity of the response function for ratings of perceived area was tested. The results show that this function is linear if the response function for magnitude estimates of perceived length is linear. A problem for future research is pinpointed.

The response function relates measures of sensory intensity to values of sensory intensity. There is evidence that this function is linear for ratings of sensory intensity (Anderson, 1981, 1982, 1996, pp. 94-96; Curtis & Fox, 1969). The present study explored whether the response function was linear for ratings of perceived area (hereafter called area) of surfaces presented frontally.

The area of a rectangle presented frontally is

$$\alpha_R = \omega\,\eta \qquad (1)$$

with ω the perceived width and η the perceived height of the rectangle.

The psychophysical function relates measures of sensory intensity to measures of physical intensity. This function is linear when it is obtained from magnitude estimates of perceived length (hereafter called length) of lines presented frontally (Baird & Vernon, 1965; Bogartz, 1979; Ekman & Junge, 1961; Fagot, 1982; Hartley, 1977, 1981; Irvin & Verrillo, 1979; Kerst & Howard, 1983; Masin & Vidotto, 1983; Pitz, 1965; Reese, Reese, Volkmann, & Corbin, 1953; Schiffman, 1965; Stevens & Galanter, 1957; Stevens & Guirao, 1963; Svenson & Åkesson, 1966; Teghtsoonian, 1965; Teghtsoonian & Beckwith, 1976; Teghtsoonian & Teghtsoonian, 1971; Verrillo, 1981, 1982, 1983; Zwislocki & Goodman, 1980). Assuming that the response function for magnitude estimates of length is linear, the linearity of the psychophysical function for length implies that

$$\omega = k_0\, w + k_1 \qquad (2)$$

and

$$\eta = k_0\, h + k_1 \qquad (3)$$

with w the measure of the physical width and h the measure of the physical height of the rectangle and with k_0 and k_1 unknown parameters.

Equations 1–3 yield

$$\alpha_R = k_0^2\, w\, h + k_0\, k_1\, (w + h) + k_1^2\,. \qquad (4)$$

Let us assume that the response function for the rating R_R of α_R is

$$R_R = c_0\, \alpha_R + c_1 \qquad (5)$$

with c_0 and c_1 unknown parameters.

Equations 4 and 5 yield

$$R_R = c_0\, k_0^2\, w\, h + c_0\, k_0\, k_1\, (w + h) + c_0\, k_1^2 + c_1\,. \tag{6}$$

Equation 6 implies the prediction that R_R varies linearly with w when h is fixed. In the experiment reported below, subjects rated the area of 13 rectangles with h fixed at 21 cm and with w varying in steps of 1.5 cm from 3 to 21 cm. The linearity of the response function for ratings of area (Equation 5) was tested by testing whether R_R varied linearly with w.

Note that this test is based on the assumption that the response function for magnitude estimates of length is linear. If the prediction that R_R varies linearly with w is verified, one concludes that the response function for ratings of area is truly linear if the response function for magnitude estimates of length is truly linear.

In the experiment reported below, subjects were asked to rate the area of 13 disks of different area. Each physical area of disks was equal to the physical area of one of the rectangles used to test Equation 5. It may easily be shown that the area of these disks was

$$\alpha_D = k_0^2\, w\, h + 2\, k_0\, k_1 \sqrt{\pi\, w\, h} + \pi\, k_1^2\,. \tag{7}$$

Consequently the rating of α_D was

$$R_D = c_0\, k_0^2\, w\, h + 2\, c_0\, k_0\, k_1 \sqrt{\pi\, w\, h} + \pi\, c_0\, k_1^2 + c_1\,. \tag{8}$$

I have calculated that the root-mean-square deviation of mean ratings of rectangle area obtained by Anderson and Cuneo (1978) from corresponding R_Rs predicted by Equation 6 is minimized when $k_1 = 0$. For h fixed, Equation 8 shows that R_D varies linearly with w if $k_1 = 0$ and varies nonlinearly with w if $k_1 \neq 0$. The possibility that $k_1 = 0$ was tested by testing whether R_D varied linearly with w.

To appraise sensitivity of ratings to nonlinearity subjects were asked to rate the length of 13 horizontal lines of different length. Each physical length of lines equaled the physical diameter of one of the disks used to test whether $k_1 = 0$. Since

$$\alpha_D = ¼\, \pi\, \delta^2 \tag{9}$$

with δ the perceived diameter of the disk, it must be that

$$\alpha_D = ¼\, \pi\, (k_0^2\, d^2 + 2\, k_0\, k_1\, d + k_1^2)\,. \tag{10}$$

with d the measure of the physical diameter of the disk. Equation 10 shows that ratings of line length must vary nonlinearly with w. Sensitivity to nonlinearity was appraised by testing this implication.

Method

Subjects

Nineteen university students participated in the experiment as subjects.

Stimuli

Experimental stimuli were achromatic rectangles, disks, or horizontal lines each with luminance of 5 cd/m^2 located in the middle of a 83 × 60 cm 25 cd/m^2 achromatic rectangular background presented frontally in the middle of the screen of a horizontal NRC PlasmaSync 50MP2 plasma monitor controlled by a Power Maintosh 7200/90 computer. Viewing distance was 270 cm. The experimental room was illuminated only by the monitor screen.

There were thirteen rectangles all with height 21 cm and with width varying in steps of 1.5 cm from 3 to 21 cm. For each rectangle there was one disk with physical area equal to that of the rectangle. For each disk there was one horizontal 1 pixel wide line with physical length equal to the physical diameter of the disk. Stimuli were shown twice randomly. To compensate for orientation each rectangle was shown once horizontally and once vertically.

Two 5 cd/m^2 achromatic standard stimuli were presented in the middle of the screen before each experimental stimulus. For rectangles or disks the standard stimuli were two squares with horizontally aligned centers, one with side length of 4 cm and one with side length of 30 cm. For horizontal lines the standard stimuli were two collinear 1 pixel wide horizontal lines, one with length of 4 cm and one with length of 30 cm. The width of the gap between the standard stimuli was 16 cm. The standard stimuli appeared for 1 sec, randomly in one of the two possible relative positions. The time between the offset of the standard stimuli and the onset of the corresponding experimental stimulus was of 1 sec. The experimental stimulus disappeared when the experimenter typed the response of the subject. Standard stimuli appeared 1 sec after this response was typed.

Procedure

The following instructions were displayed on the monitor screen and were read and commented when necessary by the experimenter: "In this experiment, you will be shown squares, rectangles, disks, and horizontal lines, one at a time. You are asked to rate how much the areas of the squares, of the rectangles, and of the disks are large and how much the lines are long. Ratings are to be expressed using the integer numbers from 10 to 100. The following are the two standard stimuli presented before each square, each rectangle, and each disk (the standard stimuli made of squares were presented once, with relative position selected randomly). The area of the smallest square is equal to 10 and the area of largest square is equal to 100. The following are the two standard stimuli presented before each line (the standard stimuli made of lines were presented once, with relative position selected randomly). The length of the shortest line is equal to 10 and the length of longest line is equal to 100. Each number assigned to the squares, rectangles, or disks must be in proportion to their area—the larger the area the larger the number—considering that the area of the smaller standard is 10 and that the area of the larger standard is 100. Each number assigned to the lines must be in proportion to their length—the longer the line the larger the number—considering that the length of the shorter standard is 10 and that the length of the longer standard is 100." A large response range and two standard stimuli, one much smaller than the smallest and one much larger than the largest experimental stimulus, were used to minimize biases (Foley, Cross, Foley, & Fox, 1983; Marks 1968; Parducci 1982; Parducci & Wedell 1986). Integers for ratings were restricted in the range 10–100 to avoid the bias due to the preference of individuals for digits (Baird & Noma, 1978, p. 109).

Main results

In Figure 1, the left and central diagrams show, respectively, the mean ratings of rectangle area and of disk area as a function of rectangle width, with width defined as above. In the right diagram, the larger dots show mean ratings of length of horizontal lines as a function of rectangle width, while the smaller dots show these mean ratings as a function of disk diameter. For each stimulus, the individual score for each subject was the mean of the two ratings the subject assigned to the stimulus.

The results for rectangles agree with previous findings (Anderson & Weiss, 1971). A least-squares straight line fits mean ratings of area of rectangles as a function of rectangle width. The linear trend was significant [$F(1,18) = 359$, $p < 0.0005$] and the quadratic trend was not significant [$F(1,18) = 1.97$]. These results confirm Equation 5.

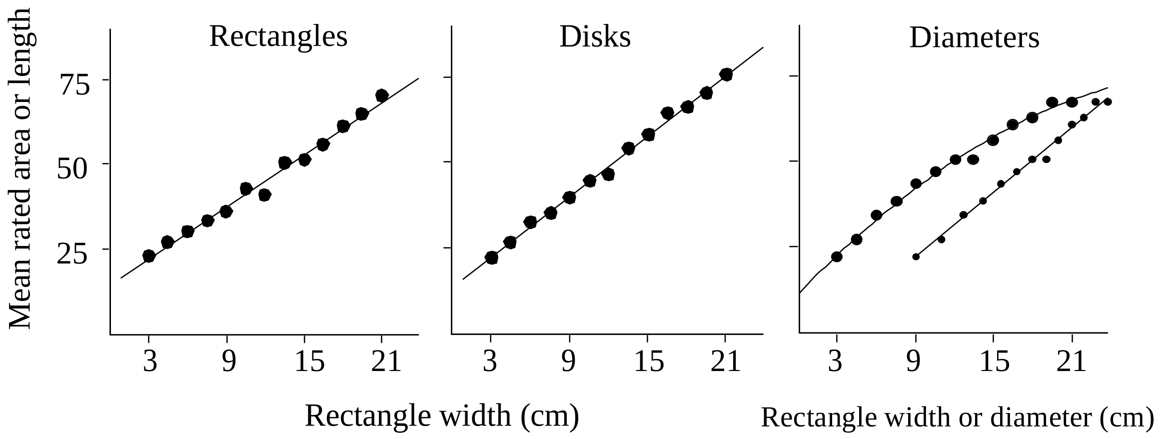

Figure 1. Mean rated area of rectangles and of disks as a function of rectangle width and mean rated length of lines as a function of rectangle width (larger dots) or of disk diameter (smaller dots).

A least-squares straight line fits mean ratings of area of disks as a function of rectangle width. The linear trend was significant [$F(1,18) = 544$, $p < 0.0005$] and the quadratic trend was not significant [$F(1,18) = 0.002$]. These results confirm that $k_1 = 0$.

In the right diagram a least-squares straight line fits mean ratings of horizontal line length as a function of disk diameter (smaller dots). This straight line shows that the psychophysical function had an exponent of 1. Stevens and Galanter (1957) found that ratings produced a psychophysical function for length with exponent 0.69 (Ward, 1974) contributing influentially to the negative view that ratings were biased. However, Stevens and Galanter (1957) used ratings without following the methodological precautions that are known today to be necessary to minimize context effects.

A least-squares parabolic arc fits the mean ratings of horizontal line length as a function of rectangle width (larger dots). The linear and quadratic trends were significant [Fs(1,18) = 302 and 19.6, respectively, $p < 0.0005$]. These results show that ratings were sensitive to nonlinearity and thus confirm Equation 5. When squared individual scores rather than individual scores were used for the statistical analyses, the linear trend was significant [$F(1,18) = 121$, $p < 0.0005$] but the quadratic trend was no longer significant [$F(1,18) = 0.41$] in conformity with the fact that disk area varied linearly with the square of the diameter (Equation 9). These results confirm that $k_1 = 0$.

Serendipetous results

A 2 (rectangle vs. disk) × 13 (rectangle width) analysis of variance showed that mean ratings of disk area were significantly higher than mean ratings of rectangle area [$F(1,18) = 11.9$, $p < 0.005$]. The mean ratings of disk area and rectangle area progressively diverged as physical area increased. The interaction was marginally significant [$F(12,216) = 1.73$, $p = 0.06$].

This finding that the shape of stimuli had an effect on rated area has no relevant implication for the line of reasoning of the present study.

It is undetermined whether the effect of shape was perceptual, mnemonic, or both. On one hand it could be that an illusory change in ω or η increasing with area caused this effect. On the other hand the following results indicate that the effect of shape could have been a memory rather then a perceptual effect. In two carefully executed experiments, Bolton (1897; see also Anastasi, 1936) had 25 subjects either select a square so that its area matched the area of a standard disk, or select a disk so that its area matched the area of a standard square. Essentially the results showed that the areas of the surfaces matched when the corresponding physical areas matched. These results indicate that the overestimation of the area of disks found in the present study could have been due to the successive comparison of experimental stimuli with remembered standard stimuli.

Conclusion

The assumption that the response function for magnitude estimates of length is linear implies Equations 2 and 3. Functional measurement and the bisection method confirm this assumption since they confirm these equations (Anderson, 1974, 1977). The present results show that the response function for ratings of area is linear if the response function for magnitude estimates of length is linear.

This conclusion leads to a problem that deserves investigation.

Since the psychophysical function for length obtained by magnitude estimation is linear and the results in Figure 1 show that the psychophysical function for length obtained by ratings is linear, magnitude estimation and ratings should both involve a linear response function for length.

The psychophysical function for area obtained by magnitude estimation is nonlinear with exponent of about 0.75 (Baird, 1970; Da Silva, Marquez, & Ruiz, 1987; Rule & Markley, 1971; Teghtsoonian, 1965; Teghtsoonian & Teghtsoonian, 1983) and the results in Figure 1 show that the psychophysical function for area obtained by ratings is linear. Thus, since the present results show that the response function for ratings of area is linear (if the response function for magnitude estimates of length is linear) it should be that the response function for magnitude estimates of area is nonlinear.

The problem is, why should the response function for magnitude estimates of area be nonlinear?

References

Anastasi, A. (1936). The estimation of area. *Journal of General Psychology*, **14**, 201-225.

Anderson, N. H. (1974). Cross-task validation of functional measurement using judgments of total magnitude. *Journal of Experimental Psychology*, **102**, 226-233.

Anderson, N. H. (1977). Failure of additivity in bisection of length. *Perception & Psychophysics*, **22**, 213-222.

Anderson, N. H. (1981). *Foundations of Information Integration Theory*. New York: Academic Press.

Anderson, N. H. (1982). *Methods of Information Integration Theory*. New York: Academic Press.

Anderson, N. H. (1996). *A functional theory of cognition*. Mahwah, NJ: Erlbaum.

Anderson, N. H., & Cuneo, D. O. (1978). The height + width rule in children's judgments of quantity. *Journal of Experimental Psychology: General*, **107**, 335-378.

Anderson, N. H., & Weiss, D. J. (1971). Test of a multiplying model for estimated area of rectangles. *American Journal of Psychology*, **84**, 543-548.

Baird, J. C. (1970). *Psychophysical analysis of visual space*. New York: Pergamon Press.

Baird, J. C., & Noma, E. (1978). *Fundamentals of scaling and psychophysics*. New York: Wiley.

Baird, J. C., & Vernon, C. W. (1965). Binary estimation. *Psychonomic Science*, **3**, 469-470.

Bogartz, R. S. (1979). Line ratio judgments yield subjective lengths proportional to physical lengths: reanalysis of Engen's data. *Perception & Psychophysics*, **26**, 247-249.

Bolton, F. A. (1897). A contribution to the study of illusions. *American Journal of Psychology*, **9**, 167-182.

Curtis, D. W., & Fox, B. E. (1969). Direct quantitative judgments of sums and a two-stage model for psychophysical judgments. *Perception & Psychophysics*, **5**, 89-93.

Da Silva, J. A., Marquez, S. L., & Ruiz, E. M. (1987). Subjective differences in exponents of psychophysical power functions for inferred, remembered, and perceived area. *Bulletin of the Psychonomic Society*, **25**, 191-194.

Ekman, G., & Junge, K. (1961). Psychophysical relations in visual perception of length, area, and volume. *Scandinavian Journal of Psychology*, **2**, 1-10.

Fagot, R. F. (1982). Constructability and the power function for line judgments. *Perception & Psychophysics*, **31**, 392-394.

Foley, H. J., Cross, D. V., Foley, A., & Fox, R. (1983). Stimulus range, number of categories, and the “virtual exponent”. *Perception & Psychophysics*, **34**, 505-512.

Hartley, A. A. (1977). Mental measurement in the magnitude estimation of length. *Journal of Experimental Psychology: Human Perception & Performance*, **3**, 622-628.

Hartley, A. A. (1981). Mental measurement of length: the role of the standard. *Journal of Experimental Psychology: Human Perception & Performance*, **7**, 309-317.

Irvin, G., & Verrillo, R. T. (1979). Absolute estimation of line length and afterimage as a function of viewing distance. *Sensory Processes*, **3**, 275-285.

Kerst, S. M., & Howard, J. H. (1983). Mental processes in magnitude estimation of length and loudness. *Bulletin of the Psychonomic Society*, **21**, 141-144.

Marks, L. E. (1968). Stimulus-range, number of categories, and form of the category-scale. *American Journal of Psychology*, **81**, 467-479.

Masin, S. C., & Vidotto, G. (1983). A magnitude estimation study of the inverted-T illusion. *Perception & Psychophysics*, **33**, 582-584.

Parducci, A. (1982). Category ratings: still more contextual effects! In B. Wegener (Ed.), *Social attitudes and psychophysical measurement*. Hillsdale, NJ: Erlbaum. (pp. 89-105).

Parducci, A., & Wedell, D. H. (1986). The category effect with rating scales: number of categories, number of stimuli, and method of presentation. *Journal of Experimental Psychology: Human Perception and Performance*, **12**, 496-516.

Pitz, G. F. (1965). Magnitude scales of line length. *Psychonomic Science*, **2**, 213-214.

Reese, E. P., Reese, T. W., Volkmann, J., & Corbin, H. H. (Eds.) (1953). *Psychophysical research, summary report 1946-1952* (Technical report No. SDC 131-1-15). South Handley, MA: U.S.N. Special Devices Center.

Rule, S. J., & Markley, R. P. (1971). Subjective differences in cross-modality matching. *Perception & Psychophysics*, **9**, 115-117.

Schiffman, H. R. (1965). Size estimation and the size of the measuring unit. *Psychonomic Science*, **3**, 479-480.

Stevens, S. S., & Galanter, E. H. (1957). Ratio scales and category scales for a dozen perceptual continua. *Journal of Experimental Psychology*, **54**, 377-411.

Stevens, S. S., & Guirao, M. (1963). Subjective scaling of length and area and the matching of length to loudness and brightness. *Journal of Experimental Psychology*, **66**, 177-186.

Svenson, O., & Åkesson, C. A. (1966). *Fractional and multiple estimates in ratio scaling* (Report No. 202). Stockholm: Psychological Laboratory, University of Stockholm.

Teghtsoonian, M. (1965). The judgment of size. *American Journal of Psychology*, **78**, 393-402.

Teghtsoonian, M., & Beckwith, J. B. (1976). Children’s size judgments when size and distance vary: is there a developmental trend to overconstancy? *Journal of Experimental Child Psychology*, **22**, 23-39.

Teghtsoonian, M., & Teghtsoonian, R. (1971). How repeatable are Stevens’s power law exponents for individual subjects? *Perception & Psychophysics*, **10**, 147-149.

Teghtsoonian, M., & Teghtsoonian, R. (1983). Consistency of individual exponents in cross-modal matching. *Perception & Psychophysics*, **33**, 203-214.

Verrillo, R. T. (1981). Absolute estimation of line length in three age groups. *Journal of Gerontology*, **36**, 625-627.

Verrillo, R. T. (1982). Absolute estimation of line length as a function of sex. *Bulletin of the Psychonomic Society*, **19**, 334-335.

Verrillo, R. T. (1983). Stability of line-length estimates using the method of absolute magnitude estimation. *Perception & Psychophysics*, **33**, 261-265.

Ward, L. M. (1974). Power functions for category judgments of duration and line length. *Perceptual and Motor Skills*, **38**, 1182.

Zwislocki, J. J., & Goodman, D. A. (1980). Absolute scaling of sensory magnitudes: a validation. *Perception & Psychophysics*, **28**, 28-38.