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## **Intuitive cognitive algebra of sliding friction**

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Intuitive physics studies our beliefs about physical events occurring in everyday life. Intuitive physics may help improve our knowledge of everyday physical events, our methods to teach physical science, and our understanding of the history of physics (McCloskey, 1983; McDermott, 1991). In four experiments we explored the beliefs about the sliding friction of a flat object on a flat horizontal surface.

### **Experiment 1**

Experiment 1 explored how individuals integrate information about the coarseness of the object and the coarseness of the horizontal surface.

#### *Method*

*Participants.* The participants were four female and six male university students with age ranging from 19 to 22 years.

*Stimuli.* Experimental stimuli consisted of two objects, one horizontal wooden disk with diameter of 4.5 cm and thickness of 1 cm with a wooden knob attached on its top (hereafter called the disk) and one  $1 \times 10 \times 18$  cm wooden board with its larger surface horizontal (hereafter called the board). Each disk weighed 45 g including the knob. Sandpaper was used to cover the top surface of the board and the bottom surface of the disk. The grit number (GN) of sandpaper was 60 or 80 or 120 or 180. There were 16 experimental stimuli, one for each combination of GN of disk and board.

One disk with GN 40 and one disk with GN 240 were used for the anchor stimuli as follows. The participant sat at a horizontal table approximately as high as the knees of the participant. Two  $10 \times 18$  cm rectangular pieces of sandpaper separated by a gap of 24 cm were attached on the table, one with GN 40 on the participant's left and one with GN 240 on the participant's right. The disk with GN 40 was placed near the left piece of sandpaper, and the disk with GN 240 was placed near the right piece of sandpa-

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per. Each of these disks with the associated piece of sandpaper attached on the table constituted the anchors stimuli.

*Procedure.* The experimental stimuli were presented as follows. The board was first placed in the middle of the anchor stimuli. While participants were holding the knob of the disk of the experimental stimulus with their preferred hand keeping the bottom surface of the disk parallel to the board, they were asked to rub twice the bottom surface of the disk and twice the top surface of the board with one finger, the index or middle.

Participants were asked to rate the amount of friction between the disk and the board while they imagined the disk sliding on the board. The integers from 10 to 100 were used for rating. The ends of the rating scale were defined as follows. Participants were asked to rub twice the piece of sandpaper and twice the bottom surface of the disk of the left anchor stimulus with one finger, the index or middle. They were told that the amount of friction due to the coarsenesses of these two surfaces was 10. Participants were then asked to rub twice the surface of the right piece of sandpaper and twice the bottom surface of the disk of the right anchor stimulus. They were told that the amount of friction due to the coarsenesses of these two surfaces was 100.

The stimuli were presented in random order once. At the end of this presentation the definition of the ends of the rating scale was repeated, and the stimuli were presented in random order once more. Before the experiment began, three stimuli selected at random were presented to familiarize the participant with the procedure.

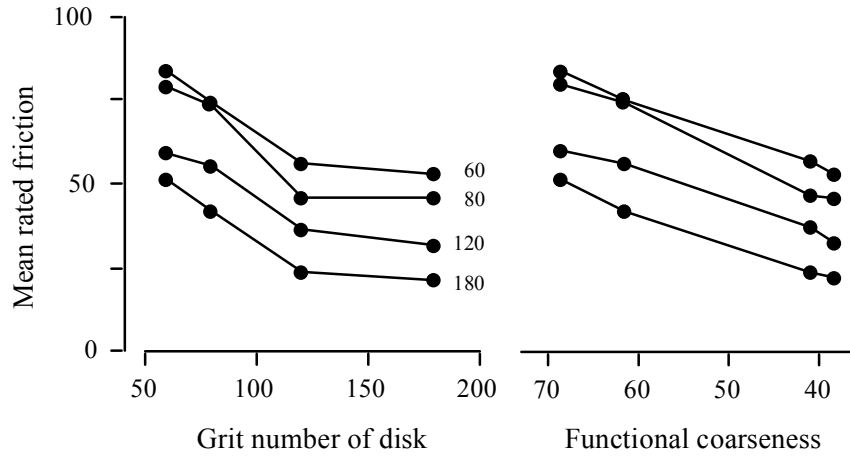
## Results

Figure 1 shows the results. In the left diagram the mean rated friction is represented as a function of GN of disk for each GN of board. Data are fit by power functions with exponents comparable to those found in previous literature (Marks & Cain, 1972; Stevens & Harper, 1962). The near-parallelism of the curves implies the additive integration

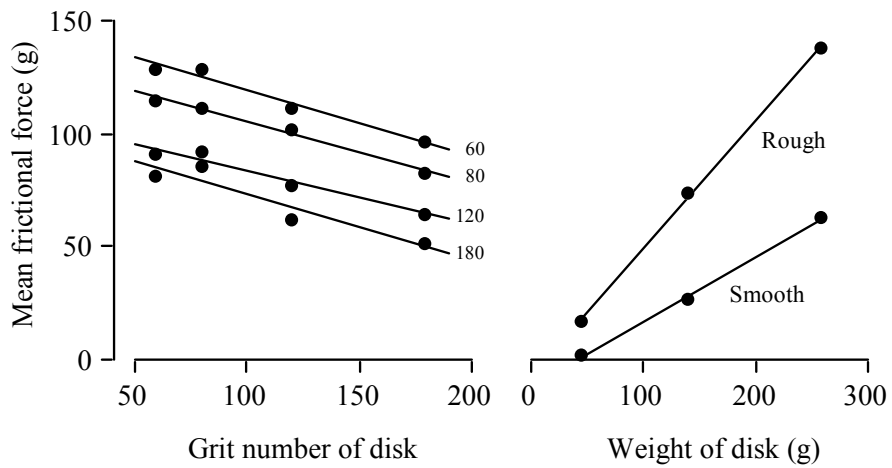
$$\textit{Imagined Friction} = \textit{Coarseness of disk} + \textit{Coarseness of board}.$$

This additive law was supported by an analysis of variance which showed that the interaction was not significant [ $F(9, 81) = 1.2$ ].

The parallelism theorem of functional measurement implies that the marginal means of GN of disk are an interval scale of perceived coarseness of disk (Anderson, 1982). The right diagram shows mean rated friction as a function of this functional coarseness of disk, for each GN of board.



**Figure 1.** Mean rated amount of friction plotted against grit number (left diagram) and functional coarseness (right diagram) of disk, for each grit number of board.



**Figure 2.** Mean static frictional force in grams plotted against grit number of disk for each grit number of board (left) and against the weight of a disk sliding on a smooth or a rough horizontal surface (right).

To determine how the law of imagined friction compares with the corresponding law of physical friction, we measured static frictional force for each combination of GN of disk and board. Measurements were made 20 times for each combination. To use the full scale of our dynamometer, the weight of each disk was increased to 140 g by placing an object on top of the knob. Mean frictional force is reported in the left diagram in Figure 2 as a function of GN of disk for each GN of board. The results show that static frictional force depended on GNs additively. This physical law is supported by the non significant interaction [ $F(9, 171) = 0.47$ ]. We may conclude that participants believed that friction depended on the coarsenesses of surfaces according to an additive law in agreement with the presently determined physical law of static frictional force.

## Experiment 2

This experiment was designed to explore how individuals integrate information about object weight and area and orientation of contact surface.

### *Method*

*Participants.* The participants were two female and eight male university students with age ranging from 19 to 25 years. None had participated in Experiment 1.

*Stimuli.* The participant sat at the table used in Experiment 1. The top surface of the table was smooth. Each stimulus was one of three boxes each measuring  $4 \times 7.7 \times 20.6$  cm. All surfaces of the stimulus were covered by smooth gray cardboard. The stimulus weight was either 0.09 or 0.92 or 2 Kg.

For each weight, the stimulus was placed on this table in one of six positions each corresponding to a different combination of area and orientation of contact surface. The area of this surface was 31 or 82 or 158 cm<sup>2</sup>.

The orientation of the contact surface was such that the longer side of the surface was either perpendicular or parallel to the frontal plane of the participant. Thus, there was a total of 18 stimuli.

*Procedure.* Participants were asked to rate the amount of force necessary to move the stimulus while they were imagining to push the stimulus with one finger. They were told to imagine the point of contact of the finger tip to be near the middle of the base of the stimulus surface facing the participant. The integers from 1 to 100 were used for rating with "1" being defined to represent the amount of force necessary to move a light box such

as an empty match box and “100” as the amount of force necessary to move a heavy box such as a box filled with bricks.

Before each rating, the experimenter put the stimulus on the upturned palms of the hands of the participant for about 1 sec. Immediately after, the stimulus was positioned on the table in front of the participant for rating. Stimuli were presented in random order once. Before the experiment began, three stimuli selected at random were presented to the participant for familiarization with the procedure.

### **Results**

The left diagram in Figure 3 shows mean rated force, averaged over the two orientations, as a function of stimulus weight for each area of contact surface. An analysis of variance showed that no factor except weight and no interaction were significant.

### **Experiment 3**

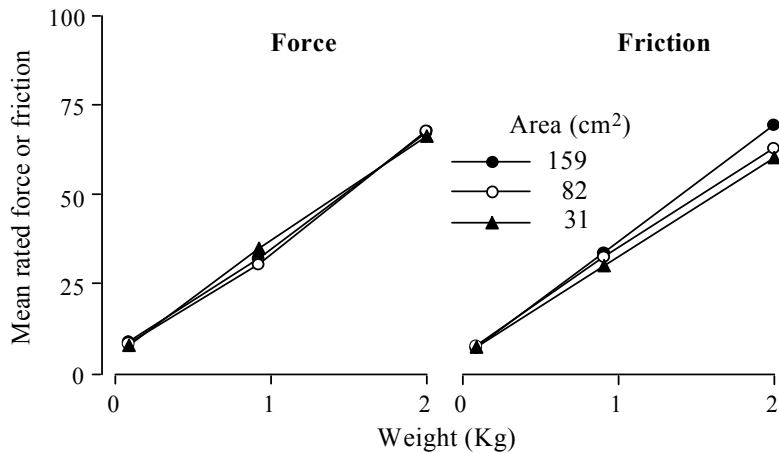
The results of Experiment 2 suggest that participants thought that area and orientation of contact surface were irrelevant for friction, in agreement with physics (Bowden & Tabor, 1958). However, participants may have assumed that stimuli were frictionless. To resolve this possibility, participants were asked to judge imagined friction rather than imagined pushing force.

#### *Method*

*Participants.* The participants were nine female and six male university students with age ranging from 20 to 25 years. None had participated in Experiments 1 and 2.

*Stimuli.* The stimuli and the presentation conditions were the same as those used for Experiment 2.

*Procedure.* The procedure was the same as that used for Experiment 2 except that participants were asked to rate the amount of friction between the contact surface of the box and the top surface of the table on which the box was placed while they were imagining that the box moved forward and backward. The ends of the rating scale were defined as follows: “1” represented the amount of friction between the table and a light box such as an empty match box and “100” represented the amount of friction between the table and a heavy box such as a box filled with bricks.



**Figure 3.** Mean rated amount of imagined force necessary to push a box on a horizontal table (left) and mean rated imagined amount of sliding friction of the same box (right) as a function of box weight for each area of the contact surface.

### Results and discussion

In the right diagram in Figure 3, mean rated friction averaged over the two orientations is shown as a function of stimulus weight. Data are fit by a fan of diverging straight lines which implies a multiplicative integration:

$$\text{Imagined Friction} = \text{Area} \times \text{Weight}.$$

An analysis of variance supported this law: area and the interaction area  $\times$  weight were significant with the linear-by-linear trend component of the interaction being the only significant one [ $F(2, 28) = 4.8, p < 0.05$ ,  $F(4, 56) = 2.6, p < 0.05$ , and  $F(1, 14) = 12.9, p < 0.005$ , respectively].

Mean rated friction averaged over area was slightly higher for the perpendicular rather than the frontal orientation for the heaviest box and was slightly lower for the other two boxes. The interaction orientation  $\times$  weight was significant [ $F(2, 28) = 3.3, p = 0.05$ ]. Orientation and all the other interactions involving orientation were not significant.

The results show that area and orientation of contact surface affected the judgment of friction. Thus, the results obtained in Experiment 2 mean that when participants judged the force necessary to move the stimulus they tacitly assumed that the stimuli were frictionless.

### Experiment 4

Experiments 2 revealed no effect of stimulus orientation and Experiment 3 showed a small effect of this factor. In Experiment 3 participants attributed a phenomenal weight to the stimulus, since they hefted the stimulus before each rating. The following experiment was made to determine whether orientation may affect the judgment of friction more strongly at a more cognitive level. The experiment was a repetition of Experiment 3 with the essential change that now the weight of the stimulus was communicated only verbally to the participant during the experiment.

#### *Method*

*Participants.* The participants were seven female and six male university students with age ranging from 20 to 42 years. None had participated in Experiments 1–3.

*Stimuli.* The stimuli and the presentation conditions were the same as those used for Experiment 3 except for the following two changes. Each of the 18 stimuli was presented once on a rectangular smooth pale-brown paper and once on a rectangular piece of dark-purple sandpaper. Both pieces of paper measured  $23 \times 27$  cm and were attached on a  $37 \times 37$  cm plywood board placed horizontally on the table. A  $57 \times 55$  cm cardboard screen was placed on the table to hide part of the table to the participant so that the operations of positioning of stimuli were invisible to the participant. Stimuli were presented to the participant by removing this screen.

*Procedure.* The procedure was the same as that used for Experiment 3 except for the following changes. The participants were given the following preliminary information. The two pieces of paper on which the experimental stimuli were going to be positioned were presented to the participants. This was done by first placing one piece of paper on the table and then removing the screen. The participants were invited to rub the fingers of their preferred hand on this piece of paper so to feel how smooth or rough it was. Subsequently, participants were asked to weigh in their hands the three boxes used in Experiment 3, in the order from the lightest to the heaviest. They were told that these boxes weighed 100 g, 1 Kg, and 2 Kg, respectively. As soon as each experimental stimulus was presented by removing the screen the experimenter told the participant which was the weight of the stimulus. Thus, in this experiment, participants only imagined the weight of the box while they imagined the box moving forward and backward.

## Results and discussion

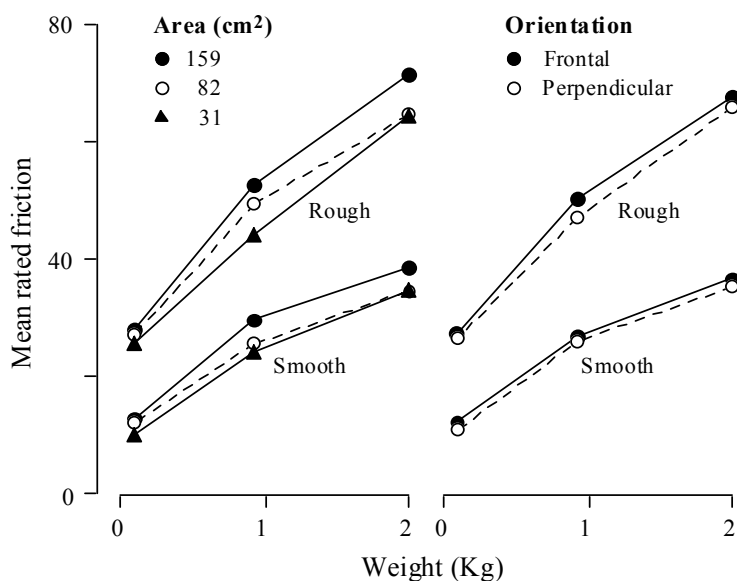
The left diagram in Figure 4 shows mean rated friction averaged over the two orientations, for each area of contact surface and for the smooth and rough horizontal surfaces. The right diagram shows mean rated friction averaged over the three areas, for each orientation of contact surface and for the smooth and rough horizontal surfaces.

The results confirm the effects of weight and coarseness on friction. The linear fan pattern supports a multiplication law:

$$\text{Imagined Friction} = \text{Coarseness} \times \text{Weight}.$$

This law was supported by an analysis of variance: all interactions involving coarseness were not significant except for the coarseness  $\times$  weight interaction with the linear-by-linear trend component being the only significant one [ $F(2, 24) = 12.7$  and  $F(1, 12) = 15.8$ ,  $p < 0.005$ , respectively].

We measured the static frictional force for the disk with GN 180 (Experiment 1) on the smooth and rough horizontal surfaces (Experiment 4) for



**Figure 4.** Mean rated amount of friction of a box, imagined to slide on a smooth or a rough horizontal surface, plotted against box weight. The results are shown for each area (left) and for two orientations (right) of the contact surface.



each of the weights 45, 140, and 260 g used for the disk. Measurements were repeated 10 times for each combination of disk weight and horizontal surface. Mean frictional force is reported in the right diagram in Figure 2 as a function of weight of disk. The divergence to the right of the lines shows that frictional force depended on weight multiplicatively. The interaction was significant [ $F(2, 18) = 23, p < 0.001$ ] with only the linear-linear trend component being significant [ $F(1, 9) = 43, p < 0.001$ ]. Plausibly, these results apply to static and kinetic friction. Participants' belief that coarseness and weight determined friction multiplicatively agrees with these data.

The interaction area  $\times$  weight [ $F(4, 48) = 2.3, p = 0.07$ ] was marginally significant supporting the law Imagined Friction = Area  $\times$  Weight. All other two-factor interactions involving area were not significant.

Orientation was significant [ $F(1, 12) = 7.0, p < 0.05$ ]. Each two-factor interaction involving orientation was not significant. Thus, imagined friction was lower when the longer side of the contact surface was parallel to the direction of motion.

### Conclusion

The present results show that our beliefs about friction of a flat object sliding on a flat horizontal surface obey the rules of cognitive algebra. The following rules have been established. (i) The coarsenesses of the surfaces in contact determine imagined friction additively. (ii) The coarseness of the horizontal surface and the weight of the object determine imagined friction multiplicatively. Both of these laws agree with the respective physical law. In contrast with Amontons's law that states that area and orientation of contact surface are irrelevant for physical friction (Bowden & Tabor, 1958), imagined friction (iii) is determined multiplicatively by object weight and area of contact surface and (iv) is lower when the longer side of the contact surface is parallel to the direction of motion. Given that beliefs about friction may differ from the corresponding physical laws, it seems important that the present results are used in the teaching of the physics of friction.

### References

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### Abstract

The cognitive algebra of the friction of a flat object sliding on a flat horizontal surface was studied. In agreement with physics, imagined friction depended additively on the coarsenesses of surfaces and depended multiplicatively on object weight and coarseness of the horizontal surface. In disagreement with physics, imagined friction depended on object weight and area of contact surface multiplicatively, and was lower when the contact surface's longer side was parallel to the direction of motion. These results are important for the teaching of the physics of friction.

### Riassunto

È stata studiata l'algebra cognitiva dell'attrito radente di un oggetto posto su di una superficie orizzontale. In accordo con la fisica, l'attrito immaginato dipende additivamente dalla ruvidezza delle superfici e dipende moltiplicativamente dal peso dell'oggetto e dalla ruvidezza della superficie orizzontale. In disaccordo con la fisica, esso dipende moltiplicativamente dal peso e dall'area della superficie di contatto, ed è minore quando il lato più lungo della superficie di contatto è parallelo alla direzione del movimento. Questi risultati sono importanti per l'insegnamento della fisica dell'attrito.

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