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Belief learning and revision studied with information integration theory

Anne Schlottmann[*] (London) & *Norman H. Anderson*[**] (San Diego)

Our beliefs change continually: They are updated over time, as we receive new information. Belief learning thus involves information integration. A long-controversial issue concerns order effects: Identical informers typically have different effects, depending on the order in which they are encountered. In the first part of this paper, we describe the model for serial belief learning developed by information integration theory (IIT; Anderson, 1981, 1982, 1996). This model can measure the evolving belief including order effects. In the second part, we compare this model with related approaches.

Serial belief learning model

The belief at position *n*, denoted by ρ_n, is a weighted average of informers up to that point, with value and weight of the informer at position *k* denoted by ψ_k and ω_k, respectively. *Value* represents the informer location on the belief dimension; *weight* represents its importance, i.e., the amount of information it contains. The initial term, $\psi_0\ \omega_0$, represents the initial belief, before receiving any informers.

$$\rho_n = \frac{\sum \psi_k\ \omega_k}{\sum \omega_k}, \tag{1}$$

where both sums run from 0 to *n*. This model can also be written in recursive anchor and adjustment form, needed if responses are revised after each informer:

$$\rho_k = (1 - \omega_k)\ \rho_{k-1} + \omega_k\ \psi_k\ . \tag{2a}$$

[*] Department of Psychology, University College London.
[**] Department of Psychology, University of California, San Diego.

Equation 1, in contrast, applies also if informers are stored separately, with a single integration at the end, as might be used if only one response is required (Anderson, 1981, p. 146). However, running adjustment is more efficient and may be used even when only a final response is made (Anderson, 1991, p. 116ff).

Proportional adjustment

If the running adjustment model of Equation 2a is re-written as

$$\rho_k - \rho_{k-1} = \omega_k (\psi_k - \rho_{k-1}) , \tag{2b}$$

it is obvious that the change in belief, $\rho_k - \rho_{k-1}$, is proportional to $\psi_k - \rho_{k-1}$, i.e., the distance between present informer and prior belief. This is a basic advantage of the serial averaging model. Proportional change is often observed empirically, but alternative models need extra assumptions to account for it.

Proportional adjustment appears in the serial learning diagrams of Figure 1, for data from a child version of the one-urn Bayesian task of decision theory. Each curve represents four successive beliefs developed from four successive informers. Children judged how much treasure was hidden in a city, its streets drawn on a set of cards (Schlottmann, 2001). They searched several streets, finding either gold or worthless black rock on the back of each card, and responded after each find. Each curve branch shows judgments for a 4-street sample of different composition, listed on the right. The diagrams may look complicated, but are worth understanding, because they illustrate important features of serial integration.

To read the diagrams, note that children's estimates increase after finding gold (+), which produces rising lines, and decrease after finding rock (–), which produces falling lines. Judgments thus reflect sample composition in a reasonable way. The two leftmost points show judgments after an initial gold or rock find. From each, two lines branch off for judgment after Sample 2, again rising for gold, falling for rock. The top curve thus shows how estimates increase with four successive gold finds; the decreasing curve branching off from it at Position 1 shows estimates when an initial gold is followed by three rocks. Such diagrams display the raw data in a step-by-step adjustment task.

The diagrams also show children's proportional adjustments: Samples consistent with the prior response produce less change than inconsistent samples. Compare, for instance, the difference in upward adjustment at Position 2 when gold is found following another gold (++) or rock (–+). Simi-

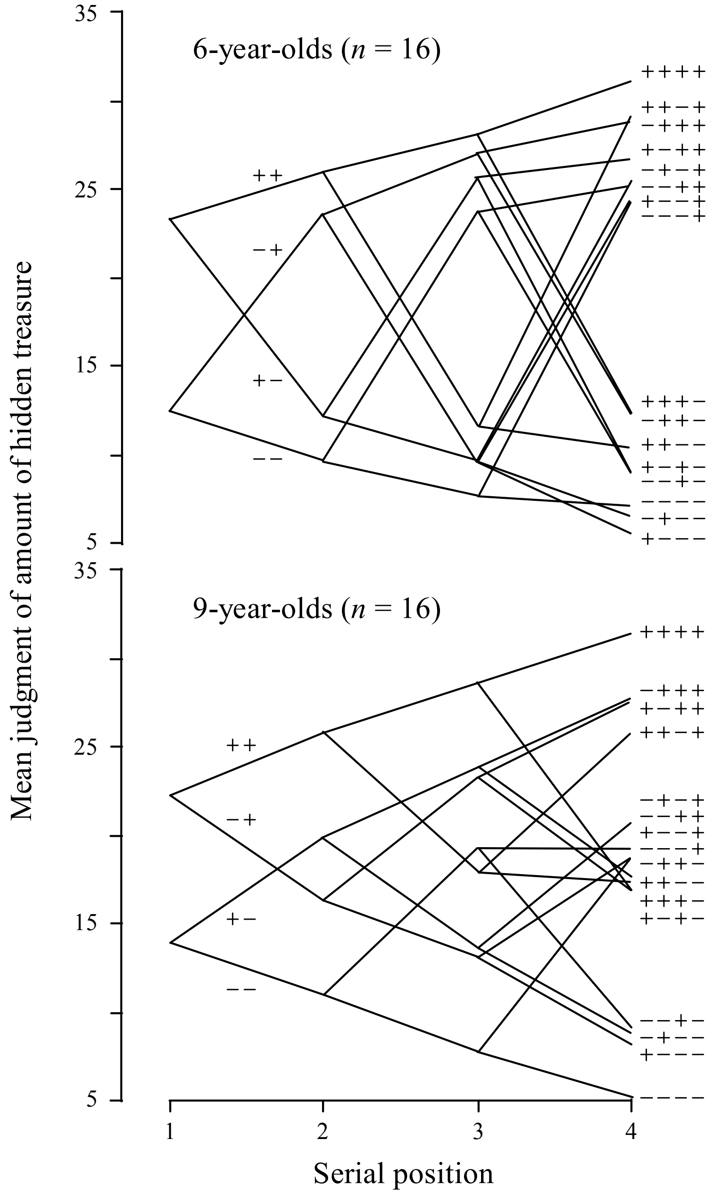

Figure 1. Serial learning curves show judgments of four samples (horizontal) of different composition (curve factor) at two ages. Proportional adjustment in the data is a natural consequence of averaging. (After Schlottmann, 2001)

lar proportional adjustment appears throughout. As already stated, the averaging model gives a natural account of such distance-proportional effects.

Order effects

Figure 1 also shows strong recency. At Position 2, responses to a –+ sample are higher than to +–. Sample composition is identical, but the order differs and the second informer has more effect. Similar recency appears at later positions.

The averaging model can represent not only recency, but all types of order effect; this is its second major advantage. These order effects depend on the weights. In Equation 1, high weights for early informers produce primacy, high weights for later informers produce recency, equal weights throughout mean no order effects. In Equation 2b, it is important to realize that the adjustment weight of the current informer, ω_k, is not typically constant, but varies with position. The weight of the prior belief, $1 - \omega_k$, increases as the sequence grows, because it is based on more and more prior informers, with concomitant decrease at each step in ω_k. If this reflects the decreasing contribution of the current informer exactly, no order effect appears; a faster decrease produces primacy, a slower decrease (or constant adjustment weight) produces recency.

Serial weight curves

The raw responses in Figure 1 confound the effect of order, reflected in informer weight, ω, and of informer content, reflected in ψ value. A third major advantage of the IIT model is that the order effect can be measured in a way that separates it from the reaction to content. This can be done for multiple serial positions, as the belief develops, even if only one response at the end is made. Thus IIT makes it possible to describe, at a high level of resolution, how belief structure evolves with incoming information.

Figure 2 shows a representation of the pure order effect, the serial weight curve, for the data in Figure 1. The R2 curves show the weights of two informers on the belief after the second informer. The second weight is higher than the first, a recency effect. The older children show less recency, a developmental trend continued in the R3 and R4 curves. These show the weights of the informers on the belief after the third and fourth informer, respectively. All curves show recency, with much higher weights for the last than for early informers.

The recency in each curve has a substantial short-term component, seen when the weight for a given sample is compared across successive beliefs. Consider, for instance, the weight for the second sample in the R2, R3 and R4 judgment. For R2, Sample 2 carries high weight, but its weight drops as soon as Sample 3 is presented. However, even the first sample carries non-zero weight, even for the youngest children. Similar serial curves, flat over early positions, with final recency, have appeared for adults (e.g., Anderson, 1996, Fig. 5.4).

As an aside, these findings with children are notable for two reasons: First, running adjustment is a natural strategy for them; second, their beliefs, contrary to popular opinion, are not tied to the perceptual presence. A previous study (Schlottmann & Anderson, 1995) reduced memory demands by giving children a scale with a movable pointer, a physical record of the belief to be revised after each sample. This initial study showed that children can use running adjustment sensibly, but not whether they, like adults,

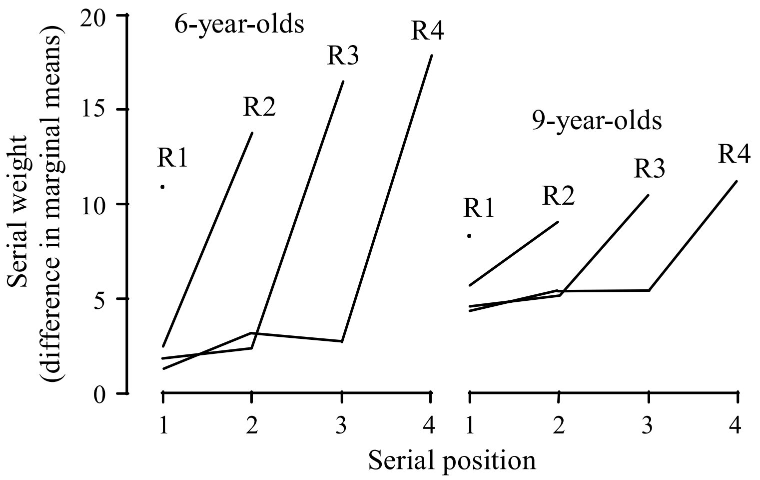

Figure 2. Serial weight curve for responses R1 to R4 in the treasure task. (After Schlottmann, 2001)

do so spontaneously. The study of Figure 2, in contrast, involved an unmarked scale, without a physical record, yet children's responses agreed reasonably well with the IIT model. Some children spontaneously left a finger on the scale as reminder of their last response, inventing a physical procedure for running adjustment. While these children generally showed less recency than those who did not point, significant effects of the earliest informers appeared throughout. The IIT approach thus showed that even if children are left to their own devices for belief revision, their beliefs go beyond the here-and-now.

Serial factor design

To obtain serial curves as in Figure 2 requires special designs. In particular, two informers are given at each position such that the difference between their values is constant and their natural weights are equal. Each position then forms a two-level factor in a serial factorial design. Subjects judge all possible combinations of factor levels in sequences with the desired number of positions. The design reflected in Figures 1 and 2, for example, involved 16 sequences in a position 1 × 2 × 3 × 4 factorial. (To reduce session length, each child saw only 8 sequences, with position 4 varied between subjects.)

The requirement of stimuli with equal value differences may seem restrictive, but has been met for a wide range of tasks. In the treasure task, for instance, children found the same amount of gold or rock in each street. Similar adult tasks often involve urns filled with different colour beads (e.g., Shanteau, 1970). In person cognition, adults judge descriptions involving different positive and negative trait adjectives, pre-tested for constant value difference (e.g., Anderson, 1973), or carefully constructed paragraphs about the target person (Anderson, 1959). Children helped Father Christmas judge story children, after learning whether each had been good or bad in previous months (Schlottmann & Anderson, 1995; Schlottmann, 2001). Stimuli with equal scale differences are often achievable and considerably simplify analysis.

Weight estimation and model tests

The weight at each position can be estimated from the corresponding main effect in the serial-factor analysis of variance: The weight is proportional to this main effect. The main effect reflects weight and value, but weight proportionality holds if the value difference is set constant by design. Thus the weights in Figure 2 were expressed as differences in marginal means.

The validity of these weight estimates depends on the validity of the averaging model itself. Model validity can also be tested with the standard analysis of variance. The model fits if the interactions of the serial positions are negligible. With a position 1 × 2 × 3 × 4 design, as in Figure 1, there are 16 such interactions, providing a stringent test of goodness of fit.

This simple analysis relies on equal value differences, as described above. However, testing model fit and estimating parameters is also possible in the general, more complex case, through use of an iterative procedure in the AVERAGE program (Zalinski & Anderson, 1991).

Determinants of primacy and recency

Of the many determinants of primacy-recency, perhaps the most important is attention. Judgments of persons described by a serial trait list show strong primacy, illustrated in the downtrend of the belief curve in Figure 3. This curve shows the serial weights provided by the model of Equation 1, i.e., weights are normalized to sum to 100.

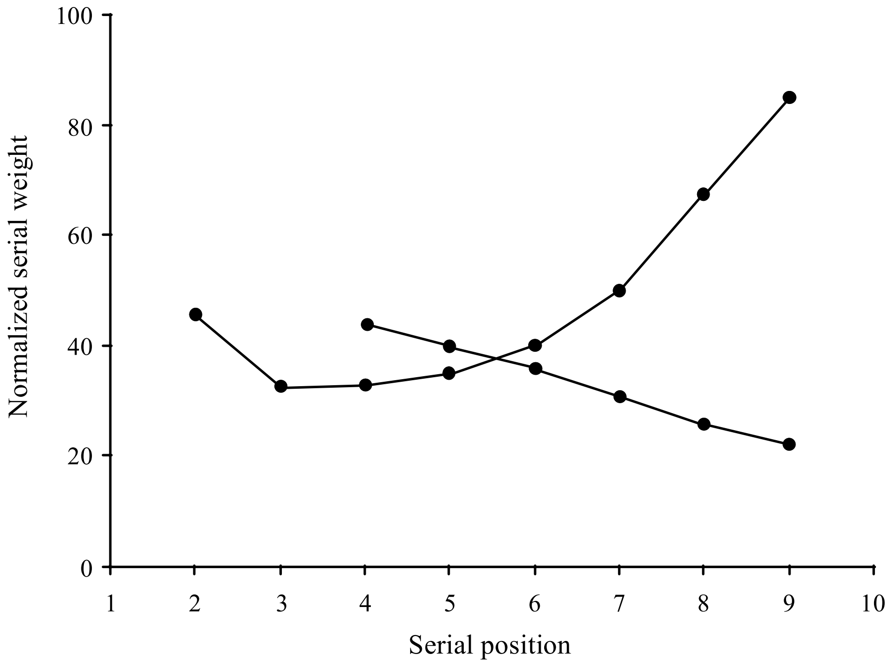

Figure 3. Serial curve for adult judgment in a final response person cognition task shows linear primacy, but recall shows recency over the same positions. (After Anderson, 1996, Figure 11.1)

The initial view was that later adjectives changed meaning to fit with the judgment based on early adjectives, but eventually the primacy was shown to result from decreased attention over positions (Anderson, 1981).

When attention decrement is prevented, for instance, by instruction to pronounce each adjective as it is presented, primacy decreases or may turn to recency. Other experiments support the attention interpretation, implying that primacy-recency is highly sensitive to experimental variations. Consider, for instance, response mode: Recency tends to appear with step-by-step responses, but when only one response at the end is made, primacy or reduced recency is more typical. In an unpublished study by the first author, the same children showing strong recency with step-by-step responses in Figure 2 showed fairly flat weight curves when only a final response was required, with children stating each seen informer out loud, in parallel to the results with adults.

Serial weight curves of belief are analogous to serial recall curves. Their different shapes, illustrated in Figure 3, underscore the importance of the belief curves. The uptrend for recall indicates strong recency, typical in memory research, in contrast to primacy for the belief curve: What is remembered best thus has least effect on the belief. Comparison of the curves implies that belief and recall involve different memory systems, a result now generally accepted (Anderson & Hubert, 1963; Hastie & Parks, 1986).

To conclude, the serial weight curves are a novel contribution of IIT. They reveal, in much detail, how the belief develops below the level of the overt response. They thus provide a high-resolution view of belief learning, highlighting the value of the model for cognitive analysis.

Comparison with other formulations

Normative theory

The goal in IIT is to describe and understand the process of belief learning, including order effects. This contrasts with the dominant, normative perspective in judgment-decision, which emphasizes optimal strategies. From the normative point of view, as in Bayesian approaches (e.g., Plach, 1999), order effects should not exist. Their frequent occurrence is a nuisance, presenting problems for analysis, a shortcoming in human decision-making competence, a bias inexplicable within normative theory itself.

The IIT perspective, in contrast, not only describes human behaviour in a detailed way, but deviations from the normative predictions appear as natural consequences of the serial process. In addition, the IIT approach allows consideration of the ecological function of order effects. The serial curves in Figure 2, and similar findings with adults, show that beliefs can have two distinct components, a surface component in immediate reaction to the current informer that declines rapidly as new information is received, and a basal component more resistant to change.

This two-component structure seems sensible for a limited-capacity processor: Much information in the environment is redundant and one would do well to settle on a belief from early observations, freeing capacity for other tasks. This corresponds to the basal component. The surface component, in contrast, can incorporate environmental shifts into the judgment. As long as these shifts are random, their representations tend to cancel or decay, but reactions to a lasting change may accumulate. The two-component mechanism can thus be seen as a temporal information filter for effective handling of minor variations in a world that is usually, but not necessarily, redundant.

Anchoring and adjustment

The notion of anchoring and adjustment became popular with Tversky and Kahneman's work (1974) on heuristics in human judgment/decision. The basic idea, that people use an initial informer as anchor, then adjust the response based on later information, is not controversial. The same idea is reflected in the IIT running adjustment model of Equation 2b (Anderson, 1996).

In addition, however, Kahneman and Tversky claimed that adjustments were typically insufficient, i.e., they argued that primacy appeared (p. 1128). No derivation of this claim was given and, as seen above, it is not true empirically. Moreover, primacy does not follow from running adjustment per se. Running adjustment can produce primacy, no order effect, or recency (oversufficient adjustment), as discussed earlier. Kahneman and Frederick's (2002) updating of the heuristics programme does not mention the notion anymore, but anchoring and adjustment continues to be much cited, in explanation of over- and under-confidence, compatibility or subadditivity effects in probability judgment (Gilovich, Griffin, & Kahneman, 2002). Without a formal theory, however, such accounts lack bite.

Traditional order effect paradigm

The traditional order effect paradigm rests on a difference score. Two informers, A and B, are given in AB and BA order, and a judgment is obtained for both. With A > B, a positive difference indicates primacy – the first informer has greater effect – a negative difference indicates recency.

In this paradigm, in effect, only two serial positions can be analysed (but A and B can, of course, be composites of multiple informers). Only a net order effect is measured in which primacy/recency cannot be localized. An effect could be due to A, B, or both together. Process questions and the temporal development of the belief cannot be addressed in any direct way. Thus the traditional paradigm has less resolution than the IIT paradigm.

Order effects were initially studied as a practical problem in, for example, (dis)advantages of being first/last in legal proceedings (e.g., Kerstholt & Jackson, 1998) or accounting (e.g., Ashton & Kennedy, 2002). The AB-BA paradigm originated from this research tradition and its main question was how to predict primacy-recency from task characteristics. However, no simple rules linking the two emerged. This points to the necessity of process analysis, for which the AB-BA paradigm is insufficient. While it is still popular in applied work, future studies could benefit from the capabilities of IIT.

Hogarth-Einhorn theory

Hogarth & Einhorn (1992) proposed a running adjustment model of the serial process explicitly tied to the AB-BA paradigm. Hence it cannot measure the effect of each serial position separately. Serial belief curves, as in Figures 2 and 3, have no meaning.

Hogarth and Einhorn used their model to link order effects to task characteristics. Their service to the field was to systematize a complex set of imperfect empirical regularities (their Tables 1 and 2). This led to differentiated order effects predictions, presented as a test of their model. However, the predictions had mixed success (Baumann & Krems, 2002) – possibly because many of them did not derive from the model itself, but from parameters and constants set to reflect particular empirical findings (Cushing, 1995)[1].

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[1] The Hogarth-Einhorn model comes in averaging and adding forms, both formally similar to the IIT model, except for a very different conceptualisation of the adjustment weight (Schlottmann & Anderson, 1995, p. 1361): In IIT, weight is a primitive parameter that depends only on serial position and is measured from the data. Conceptually, weight represents the amount of information in the stimulus; it also models the order effect, as discussed above. In the Hogarth-Einhorn model, weight is defined in terms of other model parameters, especially the prior belief; it models proportional adjustment (Hogarth & Einhorn, 1992, p. 14). It is an empirical question which concept is better.

The IIT model is validated by regular and meaningful serial curves together with successful model fits, described above. Hogarth and Einhorn argue that their model is supported by experimental test, but alternative interpretations of their data are possible. For one thing, proportional adjustment per se does not support Hogarth and Einhorn's idea of prior belief weights, because it occurs naturally un-

Non-associative learning theory

Learning is seen as largely non-associative in IIT, in sharp contrast to the standard view that we learn stimulus-response associations or contingencies. What is learned in IIT is a two-stage construction, beginning with construction of goal-oriented, psychological values from physical stimuli, with a second stage of integration of multiple values into a unitary goal-oriented response. What is learned may have entirely different character from the physical stimuli.

This non-associative nature of belief learning was shown in the dissociation between recall of the stimulus informers and the belief developed from them in Figure 3. Traditionally, memory is seen as reproductive; accuracy of recall/recognition is the primary measure. However, the belief was not based on recall. These belief learning curves illustrate the analytic power of IIT.

Capability with complex stimulus fields is a further advantage of information integration learning theory. Traditional learning studies typically deal with simple stimulus fields; stimulus-response associations are developed over repetitive trials. But realistic stimulus fields are often complex, involving verbal, visual, auditory, and other kinds of informers. The overall belief constructed can, under certain conditions, be dis-integrated to reveal the separate contribution of each informer, as in Figures 2 and 3.

Perhaps most important is a broader conceptual framework that recognizes the nature of learning in everyday cognition. Functional theory of memory/learning is broader than traditional associationist conceptions. Physical stimuli are processed to determine their implications relative to operative motivations and goals. These implications are what is important, but they may differ entirely from the stimuli themselves. What is learned is often an integral of implications of multiple stimuli. What is learned is a construction essentially different from traditional stimulus-response associations. This functional approach could be relevant to learning in schools and everyday life.

Conclusion

The present information integration model for belief learning allows us to study how beliefs evolve over time. IIT led to important insights in

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der averaging, as discussed above. If averaging can be ruled out in favour of adding, proportional adjustment could involve prior belief weights. However, Hogarth and Einhorn's evidence for adding is inconclusive. A more detailed model comparison is given in Anderson (1996, p. 358ff) and Schlottmann & Anderson (2006).

the nature of serial processing and the role and determinants of order effects, seen as embedded in the serial process. IIT also has process generality – it applies to belief formation from sequential or simultaneous informers. All in all, IIT provides a useful tool for cognitive analysis that overcomes many limitations of traditional approaches to the study of serial learning and order effects.

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Abstract

The belief learning model of information integration theory describes how beliefs develop and change with serial information. The model allows measurement of how stimulus informers presented on successive trials affect successive belief states. A complete serial curve of belief development can be obtained, even when only the final belief is known. This novel capability for measuring belief learning curves gives a high-resolution, trial-by-trial view of the evolving belief, in contrast to the less detailed measures of traditional approaches to the order effect problem in belief learning. The information integration theory model has done well in demanding tests in judgment-decision and social cognition, even with young children. This article describes the basic model, how it may be tested, how it may be used to measure order effects from the data, and how it relates to other approaches.

Riassunto

Il modello dell'apprendimento delle credenze della teoria della integrazione delle informazioni descrive come si sviluppano e cambiano le credenze con l'informazione seriale. Il modello permette di misurare come gli informatori stimolo presentati in prove successive influenzano gli stati successivi delle credenze. Una curva seriale completa dello sviluppo delle credenze si può ottenere persino quando è nota solo la credenza finale. Questa nuova capacità di misurare le curve di apprendimento delle credenze fornisce una alta risoluzione, una visione prova per prova, della credenza in evoluzione, in contrasto con le misure meno dettagliate degli approcci tradizionali al problema dell'effetto dell'ordine nell'apprendimento delle credenze. Il modello della teoria della integrazione delle informazioni ha superato bene test severi di cognizione sociale e di giudizio-decisione, anche con bambini molto giovani. Questo articolo descrive il modello di base, come esso può essere controllato sperimentalmente, come può essere usato per misurare effetti d'ordine, e quale è il suo rapporto con altri approcci.

Addresses. Anne Schlottmann, Department of Psychology, University College London, Gower Street, London WC1E 6BT, UK (a.schlottmann@ucl.ac.uk). Norman H. Anderson, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA, 92093-0109, USA (nanderson@ucsd.edu).