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Functional measurement and perceptual independence

Daniel Algom[*] (Tel-Aviv)

Functional measurement is half a century old! In the Jewish tradition, fifty is the age of composure and judiciousness (*Ethics of the Fathers*, ch. 5, 26), and acting "the justice" comes fifth in Shakespeare's seven ages of man (*As You Like It*, Act 2, Sc. 7). Equipped with this salutary faculty, we are in a position to appreciate the contribution of Functional Measurement to psychology. It is fair to say that Functional Measurement had a major impact on psychological theory and experimentation. It has been a singularly powerful tool to explicate the rules employed by humans to integrate the various sources of information in their environment. Virtually every action is multiply caused, hence integration of information is as pervasive as it is necessary. Through Norman Anderson's profound ideas and research, Functional Measurement is the "gold standard" of information integration by the organism. In this article, my goal is to examine the opposite of information integration and synthesis, namely, processes of dis-integration and analysis. They are subsumed under terms such as selective attention, perceptual independence, or separability. These processes are also vital for adaptation. Does Functional Measurement speak to these processes? Conversely, does the vast literature on selectivity and independence inform Functional Measurement?

Integration and disintegration of information

Humans are notorious at capturing covariation lurking in their environment. The integration of the pertinent information is essential to survival. Humans are driven ineluctably by information. However, humans are equally prodigious at accomplishing the opposite process of focusing on a selected aspect of their environment. By definition, each activity of everyday life requires a modicum ability to attend selectively to a certain feature of the environment, excluding other irrelevant features.

[*] Department of Psychology, Tel-Aviv University.

The upshot is that there exist an antagonism between the tendency to integrate information and the tendency to disintegrate information by focusing on a single piece of information. Both are vital for adaptation and survival, yet one activity entails *association* of information and the other *dissociation* of information. Melara and Algom (2003) identified this antagonism as a fundamental paradox of human attention processes.

Functional Measurement and many tests of selectivity and independence employ a common design in which values of two (or more) stimulus dimensions, **A** and **B**, are combined in a factorial fashion. I next discuss the ways that this fundamental design is employed in the various models.

Functional Measurement

The task for the observer is to estimate a variable, **C**, *other* than either **A** or **B**. Hence the need to integrate. Model analysis focuses on the way that the psychological values of **A** and **B** are integrated to produce **C**. Note that the observer is not informed explicitly about **A** and **B**; the observer might or might not be aware of their presence and variation. The factorial design is actually confined to the discretion of the experimenter. The processing model for a given stimulus, $\mathbf{A}_i\mathbf{B}_j$, is well known (e.g., Anderson, 1981, Figure 1.1): $\mathbf{A}_i$ and $\mathbf{B}_j$ are valuated to obtain the psychological representations a_i and b_j, respectively. These are integrated (c_{ij}) to produce $\mathbf{C}_{ij}$.

Of particular interest in the present context might be the adding model, $c_{ij} = a_i + b_j$, because, to a first impression, it implies the separability of **A** and **B** in processing. Once the unobservable values are tied together with the observable ones through (at least) monotone functions, the separability of **A** and **B** with respect to **C** may become observable thorough parallelism in the factorial plot.

We can pose the central question of this query in more concrete terms now. How does parallelism speak to the issue of independence or separability of **A** and **B**? The answer is not simple because separability is always conjoined with integration within Functional Measurement. Separability or independence of **A** and **B** means that the scale value a_i (b_j) of stimulus $\mathbf{A}_i$ ($\mathbf{B}_j$) remains constant, regardless of what other stimuli it is combined with (Anderson, 1981, pp. 18-21). This property is independent of the pertinent integration rule. For example, an adding rule of integration can operate in the *absence* of stimulus independence or separability. In this case, the stimuli interact to change one another's scale values, and then an adding-type model acts on those momentary scale values.

However, the assumption of separability or independence is vital for the derivation of parallelism. With independence in force, the scale value of each stimulus remains constant in the face of various combinations with other stimuli. Acting on such scale values with an adding-type rule of integration must produce parallelism as proved by Anderson (1981, pp. 15-17). Therefore, "the independence assumption is essential for the parallelism test" (Anderson, 1981, p. 19).

Can parallelism or its absence serve as a test of independence? Anderson (1981, p. 19) argues that it can: "observed parallelism supports the independence assumption". This might well be the case in practice, but the argument is not infallible. When the scale values of the stimuli vary due to interaction (i.e., due to the lack of independence), adding them would not generally result in parallelism. In this sense, the lack of parallelism does support violation of independence. However, the changing scale values might fortuitously cancel in a way that produces parallelism upon adding. Although the chances are slight for such an eventuality, it cannot be ruled out. The upshot is that, *in practice*, observed parallelism lends support to stimulus separability.

The independence of integration and separability applies with other rules of integration within Functional Measurement. Consider the multiplying model, which yields a radiating fan of straight lines on visual inspection. Proof of the linear fan theorem (Anderson, 1981, pp. 41-42) is also based on the independence assumption, hence, "an observed linear fan pattern supports the independence assumption" (Anderson, 1981, p. 43). In point of fact, observing a linear fan provides an even weaker test than does observed parallelism. Various combinations of changing scale values (i.e., violating independence) can produce approximations to a linear fan.

General Recognition Theory

The task for the observer is to report *both* stimulus dimensions. Based on these reports, the General Recognition Theory (Ashby & Townsend, 1986) defines an array of measures for perceptual independence and separability.

Perceptual independence holds for stimulus $\mathbf{A}_i\mathbf{B}_j$ if and only if the perceptual effects of **A** and **B** are statistically independent. Sampling independence, the empirical analogue of the unobservable concept of perceptual independence, holds in stimulus $\mathbf{A}_i\mathbf{B}_j$ if and only if the probability that both features are reported is equal to the probability that feature **A** is reported (regardless of the value reported for **B**) times the probability that feature **B** is reported (regardless of the value reported for **A**). Sampling independence is simply the theorem that the joint probability of two events equals the product of the two marginal probabilities for each event. Is not sampling independence perceptual independence itself? It is not, unless certain distributional assumptions and decisional values are met. General Recognition Theory entails a decision component (much like Signal Detection Theory), so that the overt report is determined jointly by perception and decision.

Perceptual separability concerns the effects that the components **A** and **B** have upon each other *across* different stimuli. Thus, component **A** is perceptually separable from component **B** if and only if the perceptual effect of **A** does not depend on the level of **B** that **A** is combined with. An empirical analogue is the Signal Detection Theory measure of sensitivity for one component, calculated for each level of the other component. Thus, d' can be calculated for **A** (i.e., for the discrimination between $\mathbf{A}_1$ and $\mathbf{A}_2$) for all trials in which component **B** was $\mathbf{B}_1$, and it can be calculated again for **A** for the trials in which **B** was $\mathbf{B}_2$. Subject to decision and distribution constraints, perceptual separability is supported when $d'_{\mathbf{A}|\mathbf{B}_1} = d'_{\mathbf{A}|\mathbf{B}_2}$. The same calculations can be done with respect to **B** for each value of **A**. Note that the concepts of perceptual separability and perceptual independence are logically unrelated. The former relates to the marginal perceptual effects, whereas the latter concerns the relationship between the marginal and the joint perceptual effects.

General Recognition Theory highlights the critical role and complex nature of the linkage among the stimulus scale values in Functional Measurement. What kind of independence sustains the parallelism theorem? What kind does the linear fan property? Anderson's (1981) text implicates independence, but separability might well sustain the parallelism and linear fan properties. It is clear that independence and separability cannot be tested in a rigorous fashion within Functional Measurement due to the lack of overt reporting for **A** and **B**. After all, the purview of Functional Measurement is integration, not independence. The difference in goals granted, independence is essential for major theorems of Functional Measurements to hold, and, conversely, those theorems reflect on the presence of independence. So, on a pragmatic level, certain empirical results bonding the two approaches can be expected.

Garner interference

One of the stimulus dimensions, say **A**, is specified in advance as the relevant dimension for responding. The task for the observer is then to classify **A**, as quickly as possible, into $\mathbf{A}_1$ or $\mathbf{A}_2$ while ignoring the momentary value of the irrelevant dimension, **B**. The mean reaction time (RT) in this filtering condition is compared with that in a control condition in which the observer again classifies **A**, but the irrelevant dimension **B** is held at a constant value throughout the sequence of trials (e.g., **B** is always $\mathbf{B}_1$). This condition is called the baseline condition. When performance is on a par in the filtering and the baseline conditions, the dimensions are termed *separable dimensions*. When performance is worse in the filtering condition than in the baseline condition, the dimensions are termed *integral dimensions* (Garner, 1974). The difference in performance between the baseline and filtering conditions is known as *Garner interference*. The absence of Garner interference is a hallmark of separable dimensions; its presence defines integral dimensions. Garner interference documents the failure of selectivity to the relevant dimension; it is good when the dimensions are separable, but fails when the dimensions are integral.

Results in the Garner task bear on the perceptual separability of the stimulus scale values in Functional Measurement. For example, one would expect that the absence of Garner interference with dimensions **A** and **B** appears in tandem with parallelism (or with a linear fan) of **A** and **B** in Functional Measurement. In general, one expects Garnerian separable and integral dimensions to yield different factorial patterns in Functional Measurement.

The Stroop effect

The combinations of **A** and **B** divide into congruent ($\mathbf{A}_1\mathbf{B}_1$, $\mathbf{A}_2\mathbf{B}_2$) and incongruent ($\mathbf{A}_1\mathbf{B}_2$, $\mathbf{A}_2\mathbf{B}_1$) stimuli. Suppose again that **A** is the relevant dimension for responding. The Stroop effect is then the difference in performance between congruent and incongruent combinations. The presence of the Stroop effect documents the failure of full selectivity to **A**. It shows that the observer did not ignore the momentary values of **B**, corresponding or conflicting with **A**, although these values were irrelevant to the task at hand.

The classic Stroop effect documents an asymmetric failure of selectivity: **A** (e.g., color words) intrudes on the classification of **B** (print colors), but not vice versa. In terms of the implicit scale values in Functional Measurement, the independence assumption is violated, but only in one direction. When combined with a theoretically validated integration rule, say an adding-type operation, such scale values yield unique factorial patterns. I am not aware of Functional Measurement studies with Stroop-like stimuli.

Empirical evidence

Due to its main commerce in matters of synthesis, Functional Measurement has been rarely employed to explore independence. The few positive applications yielded inconsistent results. Below, I review the results of two such attempts.

Independence and integration in the processing of pain

Algom and Edelstein (1998) used two levels of electrical current applied to the underside of the wrist and two levels of uncomfortably loud noise fed to the ears. These painful stimuli were combined in a factorial design. The authors found a large amount of Garner interference (375 ms) in the classification of shocks and another large amount (212 ms) in the classification of noise. Patently, separate sources of noxious stimulation interacted in perceptual processing.

The authors calculated Stroop effects, too, as the difference in mean RT between congruent (noise and shock both at intense or at less intense levels) and incongruent (mismatching levels of electrical and auditory pains) combinations. For shock-induced pain, the Stroop effects were 89 and 280 ms, respectively, at the baseline and the filtering conditions. For auditory pain as the relevant dimension, the respective Stroop effects were 109 and 205 ms. The Stroop analyses joined those of the Garner analyses in demonstrating that separate painful sensations interact in perceptual processing.

In a General Recognition Theory analysis, the authors used shock and noise reports by the observers to calculate values of d' for shock twice: once with noise at a constant less intense level, and once with noise at a constant intense level. The d' values for the same shocks differed at the two levels of the noise. They were 1.63 and 1.22, respectively, with noise at milder and louder levels. They similarly calculated a pair of d' values for noise with shock at less intense and intense levels. The values were 1.85 and 1.22 for shock held at lower and higher levels. Clearly, shock-induced pain was not perceptually separable from noise-induced pain, and noise-induced pain was not perceptually separable from shock-induced pain. Separability failed for both criterial dimensions.

Finally, the same stimulus dimensions were subjected to integrative judgments ("overall pain") via Functional Measurement (e.g., Algom, Raphaeli, & Cohen-Raz, 1986). The results showed the data to be consistent with an adding-type integration. The lack of a shock × tone interaction supported the observed parallelism in the pertinent factorial plots. Clearly, the conclusion based on the Functional Measurement results is incompatible with those drawn on the basis of four different tests of independence.

Separability and integration of stimuli processed along Euclidian and City-block metrics

Wilkening and Lange (1989) set out to test the relationship between the rule of integration in Functional Measurement, on the one hand, and the processing of separable and integral stimuli, on the other hand. They reasoned that the city-block metric by which separable stimuli are processed should yield parallelism on integration: “The parallel pattern is what is to be expected for separable, analytic perception” (Wilkening & Lange, 1989, p. 145). By the same reasoning, integral perception (i.e., holism or interaction between components) “should manifest itself in some obvious deviation from parallelism in the factorial graph . . . for example . . . data should show some semblance to a linear fan” (Wilkening & Lange, 1989, p. 145). Although the project by Wilkening and Lange is well taken, their predictions are not strictly justified. For instance, Anderson (1981) has shown that the assumption of independence sustains the linear fan property as strongly as it does parallelism. However, Wilkening and Lange were correct in their general hypothesis: One does expect different factorial plots for separable and integral stimuli.

To test their predictions, the authors selected prototypical separable and integral dimensions (Garner, 1974). For integral dimensions, they created stimuli that varied along hue and brightness. For separable dimensions, the stimuli varied along size and brightness. The two sets of stimulus components were subjected to integration judgments and analyzed by Functional Measurement. The results were unequivocal: All factorial plots evinced parallelism. Clearly, separability and integrality did not make a difference in the factorial plots. The same pattern, parallelism, was observed regardless of whether the stimulus scale values were constant or interacting.

Independence and its sundry failures

I conclude with an adaptation to the Functional Measurement situation of the analysis of processing interactions provided by Garner (1974). Garner’s ideas anticipated much subsequent development (Ashby & Townsend, 1986). Consider Figure 1. In this diagram, there are two inputs, the stimuli $\mathbf{A}_i$ and $\mathbf{B}_j$ (or the dimensions **A** and **B**). For each input there is an appropriate output, a_i and b_j. If the two stimuli or the two corresponding stimulus dimensions are uncorrelated, then they are (potentially) independent in processing. If the solid lines connecting each stimulus with its appropriate output are the only connections existing, then the stimuli are independent in processing, and one is dealing with separable dimensions.

Consider now one way in which independence can be violated. The vertical dashed line at the left represents interactions between the input processes. In a sense, one stimulus or one stimulus dimension cannot be processed without also processing the other stimulus or dimension. The stimuli or dimensions interact at the physical level or very early in processing. They

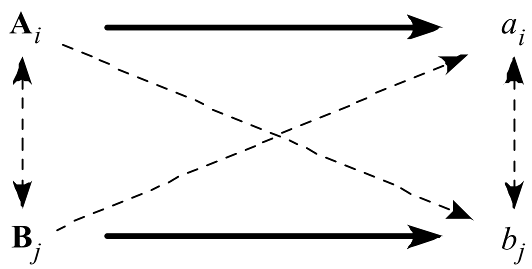

Figure 1. Process independence (solid lines) and its sundry violations (dashed lines). (After Garner, 1974, Figure 7.9)

are integral stimuli. Another implication is that the respective outputs are not, strictly speaking, sole functions of their nominal inputs.

Another violation of independence occurs when an output crosses over and responds to the inappropriate dimension or input. The crossover of the diagonals in Figure 1 represents this violation. Finally, an interaction between the outputs equivalent to that between the inputs can also occur, so that they cannot operate independently.

The reader has perhaps noticed that Figure 1 is actually the first, valuation leg of the standard Functional Measurement diagram. The current analysis, based on Garner's insights (see especially, Garner, 1974, pp. 162-164), reveals the sundry interactions that can occur already at the level of the valuation functions. Anderson is aware of this possibility, "there could be interaction effects in the valuation operation" (Anderson, 1981, p. 18), but the implications are not elaborated. They are profound.

Suppose that an experimenter varies the height and width of rectangles in a factorial design (Algom, Wolf, & Bergman, 1985; Anderson & Cuneo, 1978) and that the estimated area reveals an additive rule of integration (with children). Suppose further that a crossed-process interaction occurs (Figure 1), so that the nominal scale values for height are actually associated with width, whereas those for width are produced by physical height. The valuation operations simply cross. In this case, the factorial plots will still evince parallelism, but the designations of the axes will be incorrect.

That for width actually reflects subjective height, and the axis for height represents subjective width. The inconsistency might not exact too a high toll in this case (the integration remains the same), but it can in other more evolved situations. The upshot is that the interpretation of a particular integration result is constrained by independence or its violation at several levels along processing.

Conclusion

Integration and selection tap different underlying processes in human cognition. However, they constrain one another in experimental scrutiny. This analysis revealed significant connections between investigations of integrative and selective processes. In principle, a given integration rule operates in an invariant fashion regardless of whether the pertinent valuation functions and/or scale values interact or are independent. In practice, analysis and interpretation is more difficult, and has not been worked out in sufficient detail if the former is the case. Functional Measurement has been a formidable tool for studying integration. Incorporating perceptual independence enriches its arsenal and theoretical reach. It also explains apparent empirical puzzles (for example, when connecting Garner and Functional Measurement measures). Conversely, once the network of formal relations is explicated, Functional Measurement can serve as another, stronger tool to test independence and its locus.

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Abstract

The integration of information and the disintegration of information both are indispensable for adaptation. The former is explicated by Functional Measurement, the latter by models of selective attention and perceptual independence. To examine their relationship, a variety of selection and independence concepts is described including perceptual independence and perceptual separability within General Recognition Theory, and Stroop and Garner effects. Each of these related concepts is then connected to concepts and theorems of Functional Measurement. It is shown that the respective tools and interpretations constrain each other in significant ways. Independence informs and constraints Functional Measurement. Conversely, Functional Measurement provides a conceptual framework to understand the locus and action of independence as well as another, practical tool to assess it.

Riassunto

L'integrazione e la disintegrazione della informazione sono indispensabili per l'adattamento. La prima è spiegata dalla Misurazione Funzionale, la seconda dai modelli di attenzione selettiva e indipendenza percettiva. Per esaminare la loro relazione, vengono descritti vari concetti di selezione e indipendenza entro la Teoria Generale del Riconoscimento e gli effetti Stroop e Garner. Ciascuno di questo concetti reciprocamente collegati viene poi connesso ai concetti e teoremi della Misurazione Funzionale. Viene mostrato che i rispettivi strumenti e interpretazioni si vincolano uno con l'altro in modi significativi. L'indipendenza informa e vincola la Misurazione Funzionale. Viceversa, la Misurazione Funzionale fornisce sia uno schema concettuale per comprendere il luogo e l'azione della indipendenza che un ulteriore strumento pratico per valutarla.

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Address. Daniel Algom, Department of Psychology, Tel-Aviv University, Ramat-Aviv 69978, Israel (algomd@freud.tau.ac.il).