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The perceived magnitude of two-digit numbers: A functional measurement analysis

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How do adults perceive the magnitude of two-digit numbers? The answer to this appealingly simple question is still a mystery. At the outset, when teaching number concepts to elementary school children, two-digit numbers are introduced in an analytic fashion. The place value principle mandates clear disjunction of the component digits. The separability (Garner, 1974) of the component digits indeed is indispensable when learning about two-digit numbers. The two digits that make up the number 22 are identical yet refer to different orders of magnitude. The numbers 27 and 72 are composed of identical digits yet entail very different quantities. Once the acquisition of two-digit numbers is complete, however, people are encouraged to treat them in a unitary fashion as integral wholes (*Ganzheiten*; cf. Krueger, 1926; Wilkening & Lange, 1989). Most adults do. Two-digit numbers are usually cognized as holistic, indivisible stimuli. Language plays a role in this development from separability to integrality (the reverse of the typical trajectory from integrality to separability; Kemler, 1983). Language assigns a unique name to every two-digit number: 72 and 27 go by different names, despite the common components. Language helps to glue 7 and 2 into the single unique representation of 72. How is this process accomplished? What are its essential characteristics? Functional Measurement can provide a clue.

Integration versus composition with complex stimuli

The central concern of Functional Measurement is stimulus integration. It enables to uncover the rules by which various aspects of a stimulus are combined into a unitary sensation. The integration acts on the psychological representations of these aspects, not on the corresponding physical stimuli. However, formulations and formulae abound in the literature in

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which this crucially important point is obscured. Consider children's ratings of the area of rectangles (Anderson & Cuneo, 1978; Wolf & Algom, 1987). The addition rule found is often stated in the form, Area = Height + Width, the shorthand concealing the fact that it is the psychological values (not the physical values) of the base and height of rectangles that are being integrated into the impression of rectangle size. When people rate the likableness of a person on the basis of a pair of trait adjectives, say, highly intelligent and cold (Anderson, 1962), it is prima facie clear that what is being integrated are meaningful pieces of psychological information. This feature is less transparent with the estimations of area, but applies with equal force: What children (and adults) integrate are the psychological values of the presented height and width. Valuation transforms the physical stimuli into psychological sensations, and integration is accomplished with respect to those sensations.

The realization that what is integrated in Functional Measurement are full-fledged psychological values is of utmost importance. It means that complex stimuli whose individual components are not subject to such valuation lie outside the standard purview of Functional Measurement. With these stimuli, there does not exist integration because there are not psychological values to integrate. The stimuli are multidimensional, yet entail a single psychological value. Consider the word TABLE. It is a multidimensional stimulus composed of the letters A, B, E, L, and T. The letters are subject to the complex rules of English syntax (e.g., in English the letter U must always follow the letter Q), yet they do not assume psychological values individually. There is a meaning to TABLE but not to its constituent components.

The upshot is that there are two types of complex, multicomponential stimuli. The first class includes stimuli whose individual components assume meaning themselves. These meanings are then integrated to produce the response to the stimulus. The second class includes those stimuli whose individual components do *not* assume meaning. The components undergo fairly involved processes of organization, but meaning is conferred only upon the end-product. Such stimuli indeed are unidimensional stimuli from a psychological point of view.

Two-digit numbers: Integration or composition?

Do two-digit numbers belong in the first or the second class? They clearly are multidimensional stimuli, combinations of digits standing for decades (at the left) and units (at the right). How do people construct their psychological magnitude? One possibility entails the integration of the psychological magnitudes represented by each of the component digits. Consider the number 82. Presented with this number, the mathematical components, 80 and 2, each undergoes valuation onto psychological stimuli, and these subjective values are then integrated to produce the subjective magnitude of the number 82. The assumption that each of the components assumes psychological meaning is reasonable; after all, when 8 and 2 appear separately as single numbers, they posses magnitudes (unlike the letters of the word TABLE that do not carry meaning when standing singly). With integration of sensations in force, Functional Measurement can be applied to uncover the form of the integration.

According to an alternative view, the perception of two-digit numbers does not entail the integration of sensations. The component digits do not undergo valuation into separate psychological values, hence integration is irrelevant. The pairs of digits are subject to componential operations governed by the rules of arithmetic, but the full stimulus only carries meaning. Consider the stimulus “82”. It carries a definite subjective magnitude, but the component numbers 8 (indeed 80) and 2 do not carry psychological magnitudes themselves. On this view, 8 and 2 function precisely like the individual letters in the word TABLE.

Is the subjective magnitude of a two-digit number result of psychological integration, or result of specific, long-term learning? The two theories yield different predictions in a Functional Measurement analysis. Suppose that the decades and units are varied in a factorial design and that the observer makes a mark on a line below each (two-digit) number to indicate the subjective magnitude of the number. The responses (length in cm from the left-end of the line) are then displayed in a factorial plot. The units can be placed on the abscissa with a separate curve for each of the decades. What is the expected form of the factorial plot? The first theory predicts that the data form a set of radiating straight lines. The linear fan is the graphic manifestation of an underlying *adding-type* rule of integration acting on *interacting* (i.e., changing) scale values. The second theory predicts a set of parallel curves. The parallelism of the curves reflects the veridical acquisition of the meaning of each individual two-digit number. Let us expand on the two accounts.

If construction is accomplished by adding the internal values, then Weber’s law or one of its extensions governs the addition of these values and, consequently, the subjective magnitude of two-digit numbers. According to Weber’s law, the effect on subjective magnitude of increasing one component depends critically on the size of the other component. Thus, adding 6 to 20 (i.e., moving from 20 to 26) comprises a larger increment than adding the same number 6 to 80 (i.e., moving form 80 to 86). A constant addendum does not generate a constant increase in subjective magnitude. This is the hallmark of interaction. Generalizing the principle of diminishing marginal return across the entire set of two-digit numbers results in a factorial plot that appears as a family of radiating straight lines. Note that, in the present case, the linear fan pattern does not result from a ratio or multiplicative rule of integration. It results instead from an adding rule that acts on scale values that do not remain constant but rather interact according to Weber's law.

If, on the other hand, two-digit numbers act as integral wholes, then the meaning of each number is established by long-term learning of a constant stimulus-response mapping. The meaning of the number 82 is established in essentially the same way that the meaning of the word TABLE is. A two-digit number is a word; so is any one-digit number (but the same digits no longer act as semantic units when comprising part of multi-digit strings). The semantic network sustaining two-digit numbers is a subset of those sustaining familiar words in language in general and efficient to the same extent. It thus seems safe to assume that subjective magnitudes are a monotone function of the arithmetic magnitudes. Consequently, the factorial plot should emulate the normative rule of arithmetic construction and appear as a family of parallel lines. One can say that the normative parallelism obtains for the "wrong" reason. It obtains due to valid individual acquisition, and not because the observer is adding components in accord with the rules of arithmetic.

The following experiments tested these conflicting predictions for numbers in Arabic and verbal notations.

A functional measurement experiment

The stimuli were all the numbers between 11 and 99. The materials were white sheets of paper (size A4). The observer faced each page with the longer side on the top position. A single two-digit number in Arabic notation was printed on each page. The number appeared at the horizontal center close to the top. Close to the bottom there was a straight line running almost the entire horizontal length. Anchors at the left- and the right-end flanked the line. At the left, the number 0 appeared followed with three dots; at the right, the number 100 was printed preceded by three dots.

The task for the observer was to make a mark on the straight line at a location that corresponded with the felt magnitude of the number printed at the top. The observers were asked to provide their instantaneous, spontaneous reaction. There were 12 observers, volunteers from the Tel-Aviv University community. To make the task less tedious, each observer judged a random half of the full set of two-digit numbers. Each data point in Figure 1 represents the mean ratings of one number (in cm, measured from the left end of the line). The 81 data points are plotted as a set 9 curves, one for each decade.

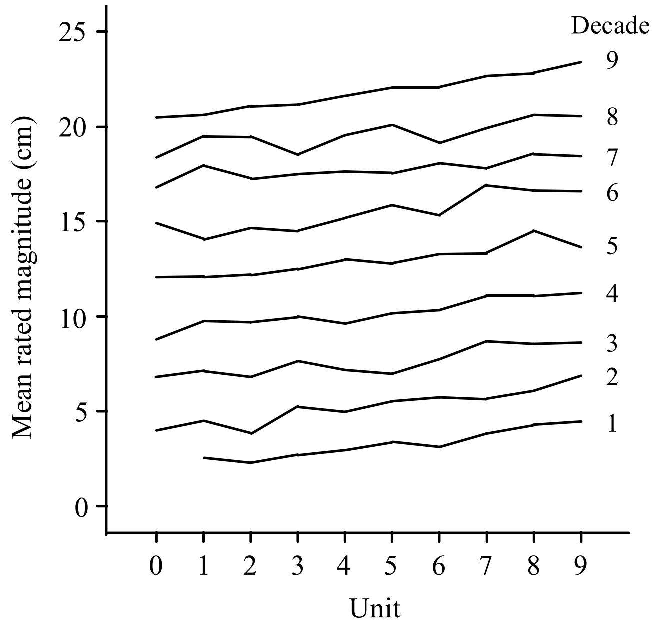

Figure 1. Mean rated magnitude evoked by two-digit Arabic numerals for each combination of unit and decade. Parallelism pattern supports unitary perception of the magnitude of two-digit numbers.

The parallelism pattern of Figure 1 is striking. The subjective magnitudes reproduced the normative arithmetic structure to a remarkable degree. Inspecting the factorial plot, it is tempting to conclude that the observer emulated normative arithmetic and obeyed an adding-type rule on constant (i.e., independent) scale values. Difficult to resist, the conclusion is wrong. Given Weber's law, an adding-type operation on interactive scale values would have resulted in an interactive, linear fan structure. The absence of this shape is interpretative. Learning to derive the magnitudes of two-digit numbers is undoubtedly a long process. Eventually, the process results in valid representations. However, the process and the resulting representations lack integration. Each representation instead is associated with a referent number in a direct, unique fashion. The reason is that two-digit numbers evolve into integral stimuli, and integrality excludes integration. As Garner has pointed out, “Integral dimensions are not separate dimensions at all, so there is no point in asking whether the organism can integrate two sources of information in such cases” (Garner, 1974, p. 152). Hence, an adding-type rule, indeed any rule of integration is gratuitous when one talks about two-digit numbers.

Two-digit numbers: Moving from Arabic into verbal notation

Does the same pattern apply when the numbers are expressed in words? The answer is not simple. The likely reason that numerals such as 8 or 2 lose their individual meanings once that they are conjoined in a two-digit number such as 82 is the confluence of meanings associated with each numeral in the string. When part of a multi-digit number, the stimulus “8” means sometimes 8 and sometimes 80 (or 800 or 8000). The solution espoused is jettisoning meaning altogether for the individual digits in multi-digit strings. This solution is not so pellucid with words. Number words do change with order of magnitude. The Arabic numeral 7 is named “seven” when single but it is named “seventy” when standing for the decades of a two-digit number. With verbal notation, one does not meet a confluence of meanings lurking under the same word. Moreover, meaning constancy has been observed with adjective descriptions of a person (Anderson, 1962), so meaning constancy might well exist for individual number words in various phrases, too. These are auspicious conditions for the operation of an adding-type rule. The following experiment tested magnitude judgments for written two-digit numbers.

The stimulus materials and procedures were the same as in the previous experiment with a single notable exception. The numbers printed at the top of each page were number words rather than Arabic numerals. Thus, the stimulus “72” from the previous experiment was replaced by the stimulus “Seventy-two” in this experiment (in the Hebrew, two-digit numbers are written as a single string of letters with the two components conjoined by the word “and”). A fresh sample of 12 observers took part in this experiment.

The resulting factorial plot is shown in Figure 2. It is a bit noisier than that in Figure 1 but it looks pretty much the same, that is, the curves are essentially parallel. How can one account for these data? Satisfaction of the following two stipulations makes true adding-type integration an attractive account for these data with words. First, number words (unlike Arabic numerals) disambiguate number names in two-digit numbers. For example, the Arabic numeral "7" is named "seven" when standing singly but it is named "seventy" when standing for the decades of a two-digit number. Second, meaning constancy might apply with the verbal components: The meaning and value of each verbal numeral remains constant regardless of what other verbal numeral it is combined with to form the two-digit number. In other words, each verbal numeral has a fixed meaning that it maintains from one two-digit combination to another. With meaning constancy in force, the parallelism of Figure 2 supports adding-type integration at the basis of the derivation of subjective magnitude of written numbers.

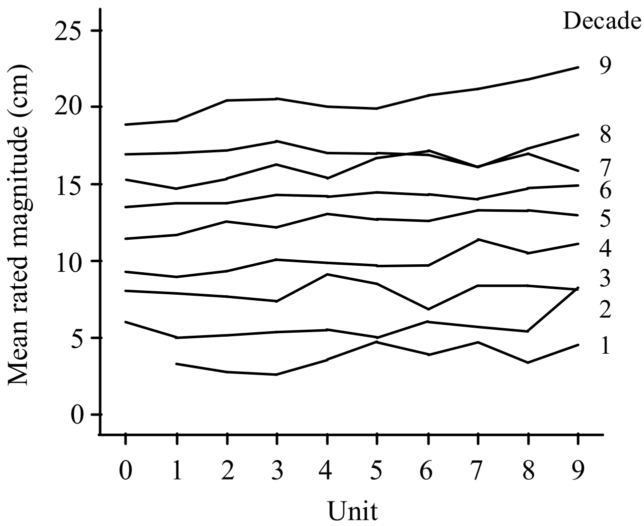

Figure 2. Mean rated magnitude evoked by two-digit numerals expressed in verbal notation for each combination of unit and decade. Parallelism pattern supports unitary perception of the magnitude of two-digit numbers.

According to an alternative account, one that we endorse, meaning constancy does not apply with number words. Meaning constancy has been shown to operate with adjective-adjective combinations in person perception (likableness), but little research exists beyond that domain. With numbers, meaning constancy violates Weber's law, a highly improbable eventuality. A sizeable literature (Stevens, 1975) shows that psychophysical laws apply to a wide range of quantitative dimensions regardless of surface representation. It is eminently reasonable to assume that number words, representing quantities in a direct fashion, do obey Weber's law. With Weber's law acting on the scale values, which in turn are subjected to an adding-type operation, a linear fan pattern is expected. Its absence argues against integration even with verbal numbers. Therefore, the parallel structure revealed in Figure 2 represents one-dimension valuation of each two-digit number. In other words, valuation and meaning is saved to "Seventy-two" and not to "Seventy" and "two" despite the fact that the latter are proper words in the language. In summary, there does not exist a difference in the construction of the magnitude of two-digit numbers across various surface representations. For both words and Arabic numerals, two-digit numbers are integral stimuli subjected to one-dimensional valuation.

Conclusion

A disproportionate amount of research in numerical cognition is devoted to single digits, but two-digit numbers and the relationship between their perception and that of single-digit numbers have been increasingly attracting attention. Single digits have been suspected to act like pictures (stimuli that cannot be named without engaging the semantic system), a mode that is improbable with two digit numbers (Damian, 2004; Fias, Reynvoet, & Brysbaert, 2001). Combinatorial operations that govern two-digit numerical perception are absent naturally from the processing of single digits. This study adds to this literature by pinpointing another feature that characterizes cognition with the two classes of numbers. Single digits that function as full-fledged semantic units lose their individual meanings once that they form parts of multi-digit numbers. The components of multi-digit numbers are subject to syntactic organization but not to semantic processing. This feature applies equally to Arabic and verbal notation of two-digit numbers. We note that Ratinckx, Brysbaert, and Fias (2005) reached the same conclusion based on experiments with masked priming. These authors also concluded that numbers (any number, single- or two-digit) are words. As words, they are subject to the laws of syntax but, like words, the whole number only engages the semantic system.

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Abstract

Are two-digit numbers perceived through direct route from the whole stimulus or through integration of their components? If the former, then two-digit numbers are merely words whose meaning (magnitude) is acquired in the same way as that of any other word in language. If the latter, then a normative adding-type rule of integration acts on scale values which change in the various combinations according to Weber's law. In two experiments, we subjected the perception of two-digit numbers to Functional Measurement analysis. The former theory predicts approximate parallelism of the factorial plot (of decades and units), the latter a linear fan shape. We found parallelism for both Arabic and verbal notations, supporting the theory that two-digit numbers are subject to a single, one-dimensional valuation.

Riassunto

Sono i numeri di due cifre percepiti per via diretta dallo stimolo globale o attraverso l'integrazione dei loro componenti? Se è vera la prima possibilità, allora i numeri di due cifre sono mere parole il cui significato (la grandezza) è acquisito allo stesso modo in cui è acquisito quello delle altre parole del linguaggio. Se è vera la seconda possibilità, allora una regola di integrazione normativa di tipo additivo agisce sui valori scalari che cambiano nelle varie combinazioni secondo la legge di Weber. In due esperimenti, abbiamo sottoposto la percezione di numeri di due cifre ad analisi tramite misurazione funzionale. La prima teoria prevede che i grafici fattoriali (delle decadi e delle unità) siano approssimativamente paralleli, la seconda prevede una configurazione a forma di ventaglio lineare. Noi abbiamo trovato parallelismo dei grafici fattoriali sia per la notazione arabica che per quella verbale, supportando così la teoria che i numeri di due cifre sono soggetti ad una singola valutazione unidimensionale.

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