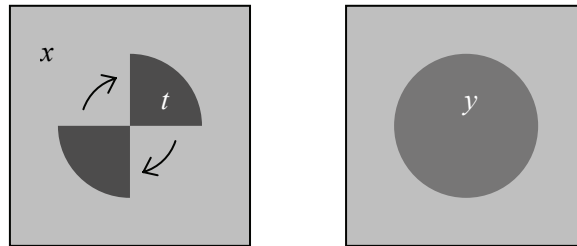


## Factorial constraints of transparency

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Metelli's (1970, 1974a, 1974b) well-known model of perceptual transparency has been confirmed by many investigators (Anderson B. L., 1997; Brill, 1984; Chen & D'Zmura, 1998; D'Zmura, Colantoni, Knoblauch, & Laget, 1997; Gerritsen, de Weert, & Wagemans, 1985; Kasrai & Kingdom, 2001). Metelli's model is based on Talbot's law of colour fusion, illustrated in Figure 1. The left diagram shows an achromatic episcotister with luminance  $t$  rotating on an achromatic background with luminance  $x$ . Curved ar-



**Figure 1.** Left: achromatic episcotister with luminance  $t$  rotating in front of an achromatic background with luminance  $x$ . Right: achromatic disk perceived when the episcotister rotates at fusion speed. The luminance,  $y$ , corresponding to the achromatic colour of the disk is expressed by Equation 1.

rows indicate rotation. The right diagram shows the uniform disk perceived in the place of the episcotister when the episcotister rotates at *fusion speed*. Talbot's law contends that the achromatic colour of this disk corresponds to the luminance

$$y = \alpha x + (1 - \alpha) t \quad (1)$$

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where  $\alpha$  is the proportion, within the disk, between the area of the open and opaque sectors of the episcotister.

In Figure 2 the left diagram shows the same episcotister in front of a bipartite achromatic background with luminances  $a$  and  $b$ . As shown in the central diagram, a transparent disk is perceived in the place of the episcotister when the episcotister rotates at fusion speed. The right diagram shows the separated left and right parts of the episcotister. According to Talbot's law, the achromatic colours of these parts correspond to the luminances

$$p = \alpha a + (1 - \alpha) t \quad (2)$$

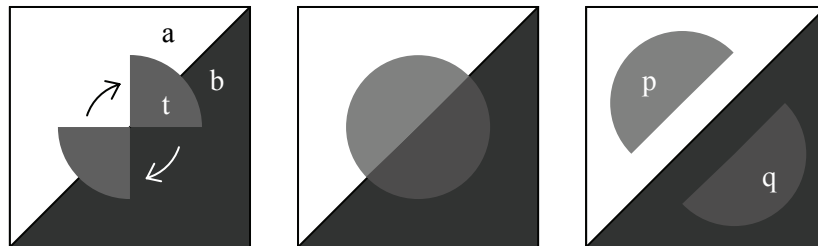
and

$$q = \alpha b + (1 - \alpha) t, \quad (3)$$

respectively. Equations 2 and 3 yield

$$\alpha = \frac{p - q}{a - b}. \quad (4)$$

Metelli (1970, 1974a, 1974b) interpreted  $\alpha$  as a physical index of the perceived extent of transparency. Figure 3 shows an example of the subjective result of varying  $\alpha$ . The background of the disk looks progressively more visible through the disk in passing from one pattern to the next, from left to right. That is,  $\alpha$  increases from left to right.



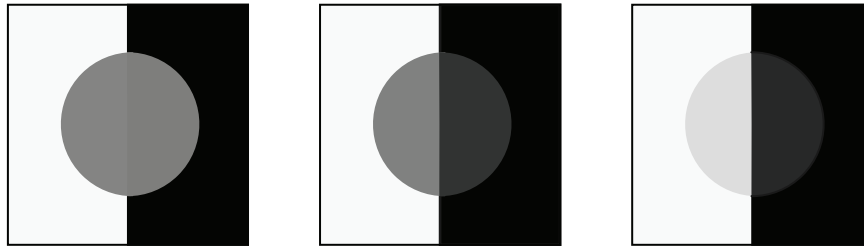
**Figure 2.** Left: achromatic episcotister with reflectance  $t$  rotating on an achromatic bipartite background with luminances  $a$  and  $b$ . Centre: transparent disk perceived when the episcotister rotates at fusion speed. Right: left and right parts of the episcotister seen in isolation; the achromatic colours of these parts correspond to the luminances  $p$  and  $q$  expressed by Equations 2 and 3, respectively.

Equation 4 plays a crucial role in Metelli's model of transparency. Since  $\alpha$  is the proportion between the area of the open and opaque sectors of the disk formed by a rotating episcotister, its value varies only between 0 (opaque sectors with total angular width of  $360^\circ$ ) and 1 (open sectors with total angular of  $360^\circ$ ). Metelli used Equation 4 to derive conditions for the occurrence of transparency. For  $\alpha$  to be included between 0 and 1, the following luminance conditions must be satisfied.

*Condition 1 (contrast reduction):*  $|p - q| < |a - b|$ .

*Condition 2 (contrast direction):*  $p < q$  if  $a < b$  (or  $p > q$  if  $a > b$ ).

Metelli (1974a, 1974b) showed empirically that transparency occurs only when these two conditions are simultaneously satisfied.



**Figure 3.** Example of varying extent of perceived transparency. The bipartite background looks progressively more visible through the disk from left to right.

Metelli's model does not specify how  $\alpha$  varies with the corresponding perceived extent of transparency. In addition to Conditions 1 and 2, a complete model for the perceived extent of transparency (rated  $\alpha$ ) must also satisfy the constraints imposed by the luminance variables that govern the perceived extent of transparency. Functional measurement analysis shows that a model is valid when it predicts the empirical pattern of data obtained by a factorial experimental design (Anderson, 1981, 1982). Thus, in addition to Conditions 1 and 2 one must consider the constraints of the pattern of factorial data. By determining how the perceived extent of transparency varies with  $p$  and  $q$  for different pairs of  $a$  and  $b$ , in the experiments reported here we have determined the factorial constraints of the empirical pattern of data that a complete model for  $\alpha$  must satisfy.

## Method

### *Participants*

The participants were thirty-eight university students. Two groups were formed, A (twenty participants) and B (eighteen participants). All had normal or corrected vision and were unaware of the purpose of the experiment.

### *Stimuli*

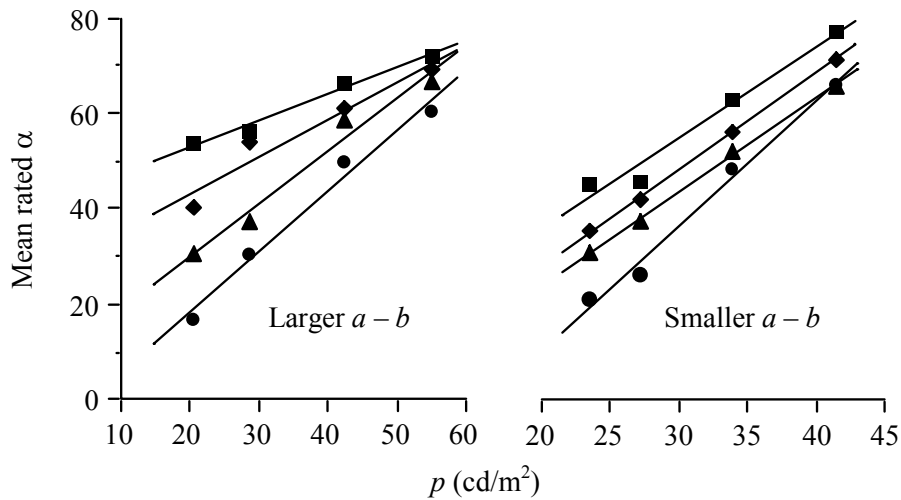
Stimuli were displayed on the  $39.5 \times 29.5$  cm screen of a CRT 19" monitor (Barco Reference Calibrator V). The luminance of the screen was  $95.8 \text{ cd/m}^2$ . Each experimental stimulus was located in the middle of the upper half of the screen. To mask possible after images, a 1-sec full-screen chessboard with  $1 \times 1$  cm light and dark grey squares ( $54.9$  and  $7.0 \text{ cd/m}^2$ , respectively) was presented before each stimulus, with the offset of the mask coinciding with the onset of the stimulus. The illumination level in the room was 1 lx. The viewing distance was 70 cm.

The experimental stimulus had the shape of the patterns shown in Figure 3. The diameter of the transparent disk was 5 cm ( $4.1^\circ$ ). The background of the disk was formed by two  $3.7 \times 7.5$  cm rectangles. Let  $a$  and  $b$  be the luminances of the left and right rectangles, respectively, and let  $p$  and  $q$  be those of the left and right halves of the disk, respectively.

For Group A, for each experimental stimulus,  $a$  and  $b$  were  $95.8$  and  $0.1 \text{ cd/m}^2$ , respectively, and there was a different combination of  $p$  of  $20.7$ ,  $28.6$ ,  $42.5$ , or  $54.9 \text{ cd/m}^2$  with  $q$  of  $2.6$ ,  $5.2$ ,  $8.8$ , or  $14.2 \text{ cd/m}^2$ .

For Group B, for each experimental stimulus,  $a$  and  $b$  were  $48.8$  and  $7.4 \text{ cd/m}^2$ , respectively, and there was a different combination of  $p$  of  $23.6$ ,  $27.2$ ,  $34.0$ , or  $41.6 \text{ cd/m}^2$  with  $q$  of  $8.8$ ,  $12.5$ ,  $16.1$ , or  $19.9 \text{ cd/m}^2$ .

Two anchor stimuli identical in shape and size to the experimental stimulus were constantly displayed, one midway between the experimental stimulus and the left side of the screen and the other midway between the experimental stimulus and the right side of the screen. For Group A,  $a$  and  $b$  were  $95.8$  and  $0.1 \text{ cd/m}^2$  in each anchor stimulus while  $p$  and  $q$  were  $20.3$  and  $18.8 \text{ cd/m}^2$  in the left anchor stimulus ( $\alpha = 0.02$ ) and were  $90.6$  and  $0.9 \text{ cd/m}^2$  in the right anchor stimulus ( $\alpha = 0.94$ ), respectively. For Group B,  $a$  and  $b$  were  $48.8$  and  $7.4 \text{ cd/m}^2$  in each anchor stimulus while  $p$  and  $q$  were  $23.6$  and  $21.8 \text{ cd/m}^2$  in the left anchor stimulus ( $\alpha = 0.04$ ) and were  $45.9$  and  $6.7 \text{ cd/m}^2$  in the right anchor stimulus ( $\alpha = 0.95$ ), respectively. The disk looked almost opaque in the left anchor stimulus and almost perfectly transparent in the right anchor stimulus.



**Figure 4.** Mean rated extent of transparency (rated  $\alpha$ ) as a function of  $p$ . Left diagram (Group A, larger difference  $a - b$ ): results for each different  $q$  of 2.6 (■), 5.2 (◆), 8.8 (▲), or 14.2 (●) cd/m<sup>2</sup>. Right diagram (Group B, smaller difference  $a - b$ ): results for each different  $q$  of 8.8 (■), 12.5 (◆), 16.1 (▲), or 19.9 (●) cd/m<sup>2</sup>.

A horizontal 10 cm long scrollbar was displayed in the middle of the lower half of the screen. Participants used the mouse of the keyboard to move a cursor left or right on the scrollbar. The cursor was a  $2.5 \times 10$  mm vertical line. Its position varied in steps of 0.5 mm.

To each group, the series of 16 experimental stimuli was presented three times consecutively, each time with stimuli in random order. The first series was used only for training.

#### *Procedure*

Participants were asked to rate the extent of transparency of the disk in each experimental stimulus. The left end of the scrollbar represented the extent of transparency of the disk in the left anchor stimulus, and the right end represented the extent of transparency of the disk in the right anchor stimulus. Participants were asked to position the cursor on the scrollbar so that its distance from the left end of the scrollbar was proportional to the extent of transparency of the disk in the experimental stimulus.

## Results

In Figure 4, the left and right diagrams show the results for Groups A and B, respectively. In each diagram, the mean rated extent of transparency is represented as a function of  $p$  for each  $q$ . Least-squares straight lines fit the data points.

For both groups, factorial curves converge upward as  $p$  increases. The the linear-by-linear trend component of the interaction was significant for both Group A [ $F(1, 19) = 42.5, p < 0.001$ ] and B [ $F(1, 17) = 5.7, p < 0.05$ ].

## Discussion

The results of Figure 4 show that a valid model for the perceived extent of transparency (rated  $\alpha$ ) must satisfy the following constraints of the factorial pattern of data when mean rated  $\alpha$  is plotted as a function of  $p$ .

*Constraint 1:* mean rated  $\alpha$  varies essentially linearly with  $p$ .

*Constraint 2:* factorial curves converge upward as  $p$  increases.

*Constraint 3:* the mean slope of factorial curves increases as the difference  $a - b$  decreases.

The input to the visual system seems to be information about the luminance steps between adjoining areas signalled by on-centre and off-centre visual neurons (Arend, Buehler, & Lockhead, 1971; Fiorentini, Baumgartner, Magnussen, Schiller, & Tomas, 1990; Krauskopf, 1963). Many authors have argued that this input consists of luminance ratios (Hess & Pretori, 1894; Jacobsen & Gilchrist, 1988; Land & McCann, 1971; Wallach, 1948). The luminance ratios between adjoining areas that were varied in the stimuli used in each experiment of this study were  $p / q$ ,  $p / a$ , and  $q / b$ . None of these ratios nor any simple arithmetical combinations of these ratios imply that factorial curves converge upward as required by Constraint 2. Thus, the present results refute the idea that simple luminance ratios or simple arithmetical combinations of these ratios are input for the mechanism of the perception of transparency.

The possible luminance differences between adjoining areas in the stimuli used in the present study were  $p - q$ ,  $p - a$ , and  $q - b$ . Taken singly, none of these differences implies that factorial curves converge upward as required by Constraint 2. Of the possible simple arithmetical combinations of these differences there is one that satisfies Constraints 1–3:

$$\text{Rated } \alpha = c_0 (p - a) (q - b) + c_1 \quad (5)$$

with  $c_0$  and  $c_1$  unknown parameters. That is, Equation 5 predicts that factorial curves are straight lines converging upward as  $p$  increases, with mean slope increasing as  $a - b$  decreases.

To apply, Equation 5 requires Metelli's Conditions 1 and 2 to be satisfied. Since Equation 5 meets Constraints 1–3, we propose Equation 5 as a possible provisional model for rated  $\alpha$ . There may exist other combinations of input information that may alternatively satisfy Constraints 1–3. Equation 5 is an example of a valid encoding of these factorial constraints.

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### Abstract

The present paper reports the results of an experiment designed to determine the constraints of the factorial pattern of data that a valid model for the perceived extent of achromatic transparency (rated  $\alpha$ ) must predict. Consider a transparent achromatic disk in the middle of two adjoining achromatic rectangles with the common border of the rectangles dividing the disk in half. Let  $a$  and  $b$  be the luminances of the left and right rectangles, respectively, and let  $p$  and  $q$  be the luminances of the left and right halves of the disk, respectively. By varying  $p$  and  $q$  factorially for different pairs of  $a$  and  $b$  and plotting mean rated  $\alpha$  as a function of  $p$ , we have found that a valid model of transparency must meet the constraints that (i) the rated extent of transparency of the disk varies essentially linearly with  $p$ , (ii) that factorial curves converge upward as  $p$  increases, and (iii) that the mean slope of factorial curves increases as the difference between  $a$  and  $b$  decreases. A new model of transparency is proposed which satisfies these constraints.

### Riassunto

Il presente articolo riporta i risultati di un esperimento eseguito per determinare i vincoli della configurazione fattoriale dei dati che un modello valido del grado percepito di trasparenza acromatica ( $\alpha$  valutato) deve prevedere. Si consideri un disco acromatico trasparente al centro di due rettangoli acromatici adiacenti il cui bordo comune divide il disco a metà. Siano  $a$  e  $b$  rispettivamente le luminanze del rettangolo sinistro e destro e siano  $p$  e  $q$  rispettivamente le luminanze della metà sinistra e destra del disco. Variando  $p$  e  $q$  fattorialmente per coppie differenti di  $a$  e  $b$  e rappresentando le valutazioni medie di  $\alpha$  in funzione di  $p$ , abbiamo trovato che un modello valido della trasparenza deve soddisfare i vincoli che (1) il grado valutato di trasparenza del disco vari essenzialmente in modo lineare con  $p$ , (2) che le curve fattoriali convergano verso l'alto all'aumentare di  $p$ , e (3) che il coefficiente angolare medio delle curve fattoriali aumenti al diminuire della differenza fra  $a$  e  $b$ . Viene proposto un nuovo modello della trasparenza che soddisfa questi vincoli.



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